

# From Farey fractions to the Klein quartic and beyond\*

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## Abstract

In a paper published in 1878/79 Klein produced his famous 14-sided polygon representing the Klein quartic, his Riemann surface of genus 3 which has  $\mathrm{PSL}(2, 7)$  as its automorphism group. The construction and method of side pairings are fairly complicated. By considering the Farey map modulo 7 we show how to obtain a fundamental polygon for Klein's surface using arithmetic. Now the side pairings are immediate and essentially the same as in Klein's paper. We also extend his work from 7 to 11 as Klein also did in a follow-up paper of 1879.

*Keywords:* Riemann surfaces, Klein quartic, regular maps, Farey tessellation, modular group, principal congruence subgroups.

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# Od Fareyjevih ulomkov do Kleinovega kvartika in naprej\*

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## Povzetek

V članku objavljenem leta 1878/79 je Klein prikazal svoj slavni 14-strani poligon, ki predstavlja t.i. Kleinov kvartik, Riemannovo ploskev z rodno enakim 3 in grupo avtomorfizmov  $PSL(2, 7)$ . Njegova konstrukcija in metoda prirejanja stranic sta precej zapleteni. Upoštevaje Fareyjev zemljevid reduciran po modulu 7 pokažemo, kako dobimo fundamentalni poligon te Kleinove ploskve z uporabo aritmetike. Zdaj so prirejanja stranic lahka in v bistvu ista kot v Kleinovem članku. Prav tako razširimo njegovo raziskavo s primera 7 na primer 11, kot je to storil tudi Klein v svojem naslednjem članku iz leta 1879.

*Ključne besede: Riemannove ploskve, Kleinov kvartik, regularni zemljevidi, Fareyjevo tlakovanje, modularna grupa, glavne kongruenčne podgrupe.*

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