

# Divergence zero quaternionic vector fields and Hamming graphs

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*Dedicated to the memory of Marjan Jerman.*

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## Abstract

We give a possible extension of the definition of quaternionic power series, partial derivatives and vector fields in the case of two (and then several) non commutative (quaternionic) variables. In this setting we also investigate the problem of describing zero functions which are not null functions in the formal sense. A connection between an analytic condition and a graph theoretic property of a subgraph of a Hamming graph is shown, namely the condition that polynomial vector field has formal divergence zero is equivalent to connectedness of subgraphs of Hamming graphs  $H(d, 2)$ . We prove that monomials in variables  $z$  and  $w$  are always linearly independent as functions only in bidegrees  $(p, 0)$ ,  $(p, 1)$ ,  $(0, q)$ ,  $(1, q)$  and  $(2, 2)$ .

*Keywords: Quaternionic power series, bidegree full functions, Hamming graph, linearly independent quaternionic monomials.*

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# Vektorska polja z divergenco nič in Hammingovi grafi

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## Povzetek

V članku je predlagana možna razširitev definicije kvaternionskih potenčnih vrst, parcialnih odvodov in vektorskih polj za dve oz. več nekomutativnih (kvaternionskih) spremenljivk. Obravnavan je problem ničelnih funkcij, ki jih popisujejo vrste, ki niso formalno ničelne. Podana je povezava med analitičnim pogojem in pogojem na podgraf Hammingovega grafa. Natančneje, polinomsko vektorsko polje ima formalno divergenco nič natančno tedaj, ko je določen podgraf Hammingovega grafa  $H(d, 2)$  povezan. Dokazano je, da so monomi v spremenljivkah  $z$  in  $w$  vedno linearno neodvisni kot funkcije samo v bistopnjah  $(p, 0)$ ,  $(p, 1)$ ,  $(0, q)$ ,  $(1, q)$  in  $(2, 2)$ .

*Ključne besede: Kvaternionske potenčne vrste, bistopenjsko polne funkcije, Hammingov graf, linearno neodvisni kvaternionski monomi.*

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