

Properties of double Roman domination on cardinal products of graphs*

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Abstract

Double Roman domination is a stronger version of Roman domination that doubles the protection. The areas now have 0, 1, 2 or 3 legions. Every attacked area needs 2 legions for its defence, either their own, or borrowed from 1 or 2 neighbouring areas, which still have to keep at least 1 legion to themselves. The minimal number of legions in all areas together is equal to the double Roman domination number.

In this paper we determine an upper bound and a lower bound for double Roman domination numbers on cardinal product of any two graphs. Also we determine the exact values of double Roman domination numbers on $P_2 \times G$ (for many types of graph G). Also, the double Roman domination number is found for $P_2 \times P_n$, $P_3 \times P_n$, $P_4 \times P_n$, while upper and lower bounds are given for $P_5 \times P_n$ and $P_6 \times P_n$.

Finally, we will give a case study to determine the efficiency of double protection. We will compare double Roman domination versus Roman domination by running a simulation of a battle.

Keywords: Roman domination, double Roman domination, cardinal products of graphs, paths, cycles.

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Lastnosti dvojne rimljanske dominacije na kardinalnih produktih grafov*

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Povzetek

Dvojna rimljanska dominacija je krepkejša verzija rimljanske dominacije, ki podvaja zaščito. Območja imajo zdaj 0, 1, 2 ali 3 legije. Vsako napadeno območje potrebuje 2 legiji za svojo obrambo, in to bodisi lastni bodisi izposojeni od 1 ali 2 sosednih območij, ki pa morata še vedno ohraniti najmanj 1 legijo zase. Minimalno število legij na vseh območjih skupaj je enako dvojnemu rimljanskemu dominacijskemu številu.

V tem članku določimo zgornjo mejo in spodnjo mejo za dvojno rimljansko dominacijska števila kardinalnih produktov poljubnih dveh grafov. Prav tako določimo natančne vrednosti dvojno rimljanskih dominacijskih števil grafov $P_2 \times G$ (za mnoge vrste grafa G). Prav tako najdemo dvojno rimljansko dominacijsko število grafov $P_2 \times P_n$, $P_3 \times P_n$, $P_4 \times P_n$, za grafa $P_5 \times P_n$ in $P_6 \times P_n$ pa predstavimo zgornji in spodnji meji.

Nazadnje predstavimo študijo primera za določitev učinkovitosti dvojne zaščite. Primerjali bomo dvojno rimljansko dominacijo in rimljansko dominacijo na ta način, da bomo pognali simulacijo bitke.

Ključne besede: Rimljanska dominacija, dvojna rimljanska dominacija, kardinalni produkti grafov, poti, cikli.

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