


# Simultaneous current graph constructions for minimum triangulations and complete graph embeddings

Timothy Sun \* *Columbia University, New York, NY, USA*

Received 15 May 2019, accepted 18 January 2020, published online 21 October 2020

---

## Abstract

The problems of calculating the genus of the complete graphs and of finding a minimum triangulation for each surface were both solved using the theory of current graphs, and each of them divided into twelve different cases, depending on the residue modulo 12 of the number of vertices. Cases 8 and 11 were of particular difficulty for both problems, with multiple families of current graphs developed to solve these cases. We solve these cases, in addition to Cases 6 and 9, in a unified manner, greatly simplifying previous constructions by Ringel, Youngs, Guy, and Jungerman. All these new constructions are index 3 current graphs sharing nearly all of the structure of the simple solution for Case 5 of the Map Color Theorem.

*Keywords:* Topological graph theory, current graphs, map coloring, triangulations.

*Math. Subj. Class.:* 05C10, 05C15


---

---

\*The author was partially supported by NSF grants CCF-1420349, CCF-1563155, and CCF-1703925.

*E-mail address:* [timothysun@sfsu.edu](mailto:timothysun@sfsu.edu) (Timothy Sun)

# Simultane konstrukcije tokovnih grafov, uporabnih za minimalne triangulacije in vložitve polnih grafov

Timothy Sun \* 

*Columbia University, New York, NY, USA*

Prejeto 15. maja 2019, sprejeto 18. januarja 2020, objavljeno na spletu 21. oktobra 2020

---

## Povzetek

Problema računanja rodu polnih grafov in iskanja minimalne triangulacije za vsako ploskev sta bila oba rešena s pomočjo teorije tokovnih grafov, pri čemer se je vsak od njiju razdelil na dvanajst različnih primerov, glede na ostanek števila točk po modulu 12. Primera 8 in 11 sta bila še posebej težavna pri obeh problemih, in za njuno rešitev so razvili številne družine tokovnih grafov. V tem članku rešimo ta dva primera, poleg tega pa še primera 6 in 9, na enak način, ki precej poenostavi predhodne konstrukcije Ringela, Youngsa, Guya in Jungermana. Vse te nove konstrukcije so tokovni grafi indeksa 3 in si delijo skoraj vso strukturo z enostavno rešitvijo primera 5 izreka o barvanju zemljevidov.

*Ključne besede:* Topološka teorija grafov, tokovni grafi, barvanje grafov, triangulacije.

*Math. Subj. Class.:* 05C10, 05C15

---

---

\* Avtor je bil delno podprt s strani NSF dotacij CCF-1420349, CCF-1563155 in CCF-1703925.  
*E-poštni naslov:* [timothysun@sfsu.edu](mailto:timothysun@sfsu.edu) (Timothy Sun)