

# The cubical matching complex revisited\*

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## Abstract

Ehrenborg noted that all tilings of a bipartite planar graph are encoded by its cubical matching complex and claimed that this complex is collapsible. We point out to an oversight in his proof and explain why these complexes can be the disjoint union of two or more collapsible complexes. We also prove that all links in these complexes are suspensions up to homotopy. Furthermore, we extend the definition of a cubical matching complex to planar graphs that are not necessarily bipartite, and show that these complexes are either contractible or a disjoint union of contractible complexes. For a simple connected region that can be tiled with dominoes ( $2 \times 1$  and  $1 \times 2$ ) and  $2 \times 2$  squares, let  $f_i$  denote the number of tilings with exactly  $i$  squares. We prove that  $f_0 - f_1 + f_2 - f_3 + \dots = 1$  (established by Ehrenborg) is the only linear relation for the numbers  $f_i$ .

*Keywords:* Domino tilings, independence complexes, matching, cubical complexes.

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# Na novo obravnavani kubični kompleksi prirejanj\*

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## Povzetek

Ehrenborg je opazil, da se dajo vsa tlakovanja dvodelnega ravninskega grafa zakodirati z njegovim kubičnim kompleksom prirejanj in zmotno trdil, da je ta kompleks se-sedljiv. Opozorimo na napako v njegovem dokazu in razložimo, zakaj so ti kompleksi lahko disjunktne unije dveh ali več sesedljivih kompleksov. Dokažemo tudi, da so vsi spoji v teh kompleksih suspenzije, do homotopije natančno. V nadaljevanju razširimo definicijo kubičnega kompleksa prirejanj na ravninske grafe, ki niso nujno dvodelni, ter pokažemo, da so ti kompleksi bodisi skrčljivi bodisi disjunktne unije skrčljivih kompleksov. Za enostavno povezano območje, ki se ga da tlakovati z dominami ( $2 \times 1$  in  $1 \times 2$ ) in  $2 \times 2$  kvadrati, naj  $f_i$  označuje število tlakovanj z natančno  $i$  kvadrati. Dokažemo, da je  $f_0 - f_1 + f_2 - f_3 + \dots = 1$  (kar je pokazal Ehrenborg) edina linearna relacija za števila  $f_i$ .

*Ključne besede: Domino tlakovanja, neodvisnostni kompleksi, prirejanja, kubični kompleksi.*

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