

Distance-regular Cayley graphs with small valency*

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Abstract

We consider the problem of which distance-regular graphs with small valency are Cayley graphs. We determine the distance-regular Cayley graphs with valency at most 4, the Cayley graphs among the distance-regular graphs with known putative intersection arrays for valency 5, and the Cayley graphs among all distance-regular graphs with girth 3 and valency 6 or 7. We obtain that the incidence graphs of Desarguesian affine planes minus a parallel class of lines are Cayley graphs. We show that the incidence graphs of the known generalized hexagons are not Cayley graphs, and neither are some other distance-regular graphs that come from small generalized quadrangles or hexagons. Among some “exceptional” distance-regular graphs with small valency, we find that the Armanios-Wells graph and the Klein graph are Cayley graphs.

Keywords: Cayley graph, distance-regular graph.

Math. Subj. Class.: 05E30

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Razdaljno-regularni Cayleyjevi grafi z majhno valenco*

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Povzetek

Problem, ki ga obravnavamo, sprašuje, kateri razdaljno-regularni grafi z majhno valenco so Cayleyjevi grafi. Poiščemo Cayleyjeve grafe med razdaljno-regularnimi grafi z valenco kvečjemu 4, med razdaljno-regularnimi grafi za znane domnevne presečne tabele z valenco 5 in med vsemi razdaljno-regularnimi grafi z ožino 3 in valenco 6 ali 7. Ugotovimo, da so incidenčni grafi Desarguesovih afinih ravnin brez enega razreda vzporednih premic Cayleyjevi grafi. Pokažemo, da incidenčni grafi znanih posplošenih šestkotnikov niso Cayleyjevi grafi; prav tako niso Cayleyjevi nekateri drugi razdaljno-regularni grafi, ki izhajajo iz majhnih posplošenih štirikotnikov ali šestkotnikov. Med “izjemnimi” razdaljno-regularnimi grafi z majhno valenco smo našli dva, ki sta Cayleyjeva, in sicer sta to Armanios-Wellsov graf in Kleinov graf.

Ključne besede: Cayleyjev graf, razdaljno-regularen graf.

Math. Subj. Class.: 05E30

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