

On the general position problem on Kneser graphs*

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Abstract

In a graph G , a *geodesic* between two vertices x and y is a shortest path connecting x to y . A subset S of the vertices of G is *in general position* if no vertex of S lies on any geodesic between two other vertices of S . The size of a largest set of vertices in general position is the *general position number* that we denote by $gp(G)$. Recently, Ghorbani et al. proved that for any k if $n \geq k^3 - k^2 + 2k - 2$, then $gp(Kn_{n,k}) = \binom{n-1}{k-1}$, where $Kn_{n,k}$ denotes the Kneser graph. We improve on their result and show that the same conclusion holds for $n \geq 2.5k - 0.5$ and this bound is best possible. Our main tools are a result on cross-intersecting families and a slight generalization of Bollobás's inequality on intersecting set pair systems.

Keywords: General position problem, Kneser graphs, intersection theorems.

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O problemu splošne lege na Kneserjevih grafih*

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Povzetek

Geodetka v grafu G med točkama x in y je najkrajša pot, ki povezuje x z y . Podmnožica S točk grafa G je *v splošni legi*, če nobena točka množice S ne leži na nobeni geodetki med dvema drugima točkama množice S . Velikost največje množice točk v splošni legi imenujemo *splošnost* in označimo z $gp(G)$. Nedavno so Ghorbani in dr. dokazali, da za vsak k , ki ustreza pogoju $n \geq k^3 - k^2 + 2k - 2$, velja $gp(Kn_{n,k}) = \binom{n-1}{k-1}$, kjer $Kn_{n,k}$ označuje Kneserjev graf. V tem članku izboljšamo njihov rezultat in dokažemo, da velja isti zaključek ob predpostavki $n \geq 2.5k - 0.5$, in da je ta meja najboljša možna. Naši glavni orodji sta rezultat o križno sekajočih se družinah in rahla posplošitev Bollobásove neenakosti o presečni množici parnih sistemov.

Ključne besede: Problem splošne lege, Kneserjevi grafi, presečni izreki.

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