


Notes on weak-odd edge colorings of digraphs*

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Abstract

A weak-odd edge coloring of a general digraph D is a (not necessarily proper) coloring of its edges such that for each vertex $v \in V(D)$ at least one color c satisfies the following conditions: if $d_D^-(v) > 0$ then c appears an odd number of times on the incoming edges at v ; and if $d_D^+(v) > 0$ then c appears an odd number of times on the outgoing edges at v . The minimum number of colors sufficient for a weak-odd edge coloring of D is the weak-odd chromatic index, denoted $\chi'_{\text{wo}}(D)$. It is known that $\chi'_{\text{wo}}(D) \leq 3$ for every digraph D , and that the bound is sharp. In this article we show that the weak-odd chromatic index can be determined in polynomial time. Restricting to edge colorings of D with at most two colors, the minimum number of vertices $v \in V(D)$ for which no color c satisfies the above conditions is the defect of D , denoted $\text{def}(D)$. Surprisingly, it turns out that the problem of determining the defect of digraphs is (polynomially) equivalent to the problem of finding the matching number of simple graphs. Moreover, we characterize the classes of associated digraphs and tournaments in terms of the weak-odd chromatic index and the defect.

Keywords: Digraph, weak-odd edge coloring, weak-odd chromatic index, defective coloring, tournament.

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O šibko-lihkih barvanjih povezav v digrafih*

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Povzetek

Šibko-liho barvanje povezav splošnega digrafa D je (ne nujno pravilno) barvanje njegovih povezav, pri katerem za vsako vozlišče $v \in V(D)$ najmanj ena barva c zadošča naslednjim pogojem: če $d_D^-(v) > 0$, potem c nastopa liho-krat na vhodnih povezavah vozlišča v ; in če $d_D^+(v) > 0$, potem c nastopa liho-krat na izhodnih povezavah vozlišča v . Minimalno število barv, zadostnih za šibko-liho barvanje povezav digrafa D , je šibko-lihi kromatski indeks, označen z $\chi'_{\text{wo}}(D)$. Znano je, da velja $\chi'_{\text{wo}}(D) \leq 3$ za vsak digraf D , ter da je ta meja natančna. V tem članku dokažemo, da se da šibko-lihi kromatski indeks določiti v polinomskem času. Če se omejimo na barvanja povezav digrafa D s kvečjemu dvema barvama, potem je minimalno število vozlišč $v \in V(D)$, za katere nobena barva c ne zadošča zgornjim pogojem, defekt digrafa D , označen z $\text{def}(D)$. Presenetljivo, izkaže se, da je problem določitve defekta digrafov (polinomsko) ekvivalenten problemu določitve števila prirejanj enostavnih grafov. Poleg tega, karakteriziramo razrede pridruženih digrafov in turnirjev glede na šibko-lihi kromatski indeks in defekt.

Ključne besede: Digraf, šibko-liho barvanje povezav, šibko-lihi kromatski indeks, defektno barvanje, turnir.

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