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On an annihilation number conjecture*

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Abstract

Let $\alpha(G)$ denote the cardinality of a maximum independent set, while $\mu(G)$ be the size of a maximum matching in the graph $G = (V, E)$. If $\alpha(G) + \mu(G) = |V|$, then G is a *König-Egerváry graph*. If $d_1 \leq d_2 \leq \cdots \leq d_n$ is the degree sequence of G, then the *annihilation number* $a(G)$ of G is the largest integer k such that $\sum_{i=1}^{k} d_i \leq |E|$. A set $A \subseteq V$ satisfying $\sum_{v \in A} \deg(v) \leq |E|$ is an *annihilation set*; if, in addition, $\deg(x) + \sum_{v \in A} \deg(v) > |E|$, for every vertex $x \in V(G) - A$, then A is a *maximal annihilation set* in G.

In 2011, Larson and Pepper conjectured that the following assertions are equivalent:

(i) $\alpha(G) = a(G)$;

(ii) G is a König-Egerváry graph and every maximum independent set is a maximal annihilating set.

It turns out that the implication "(i) $\Longrightarrow$ (ii)" is correct.

In this paper, we show that the opposite direction is not valid, by providing a series of generic counterexamples.

Keywords: Maximum independent set, maximum matching, König-Egerváry graph, annihilation set, annihilation number.

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O domnevi o pragu grafa*

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Povzetek

Naj $\alpha(G)$ označuje moč maksimalne neodvisne množice, $\mu(G)$ pa velikost maksimalnega prirejanja v grafu $G = (V, E)$. Če je $\alpha(G) + \mu(G) = |V|$, potem je G *König-Egerváryjev graf*. Če je $d_1 \leq d_2 \leq \cdots \leq d_n$ stopenjsko zaporedje grafa G, potem je *prag* $a\,(G)$ grafa G največje celo število k, pri katerem je $\sum_{i=1}^{k} d_i \leq |E|$. Množica $A \subseteq V$, ki zadošča pogoju $\sum_{v \in A} \deg(v) \leq |E|$, se imenuje *množica s pragom*; če velja poleg tega $\deg(x) + \sum_{v \in A} \deg(v) > |E|$ za vsako točko $x \in V(G) - A$, potem je A *maksimalna množica s pragom* v G.

Leta 2011 sta Larson in Pepper postavila domnevo o enakovrednosti naslednjih trditev:

(i) $\alpha\,(G) = a\,(G)$;

(ii) G je König-Egerváryjev graf in vsaka maksimalna neodvisna množica je maksimalna množica s pragom.

Izkazalo se je, da je implikacija "(i) $\Longrightarrow$ (ii)" pravilna.

V tem članku dokažemo, da obratna implikacija ne velja, prikažemo pa tudi vrsto generičnih protiprimerov.

Ključne besede: Maksimalna neodvisna množica, maksimalno prirejanje, König-Egerváryjev graf, množica s pragom, prag.

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