

Semigroups with fixed multiplicity and embedding dimension*

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Received 15 February 2019, accepted 11 September 2019, published online 4 November 2019

Abstract

Given $m \in \mathbb{N}$, a numerical semigroup with multiplicity m is called a packed numerical semigroup if its minimal generating set is included in $\{m, m + 1, \dots, 2m - 1\}$. In this work, packed numerical semigroups are used to build the set of numerical semigroups with a given multiplicity and embedding dimension, and to create a partition of this set. Wilf's conjecture is verified in the tree associated to some packed numerical semigroups. Furthermore, given two positive integers m and e , some algorithms for computing the minimal Frobenius number and minimal genus of the set of numerical semigroups with

*The authors would like to thank the referee and the editor for their useful comments and suggestions to help improving this paper.

†Partially supported by MTM2017-84890-P and by Junta de Andalucía group FQM-366.

‡Partially supported by MTM2017-84890-P and by Junta de Andalucía group FQM-298.

§Partially supported by MTM2017-84890-P and by Junta de Andalucía group FQM-343.

¶Partially supported by MTM2015-65764-C3-1-P (MINECO/FEDER, UE), by MTM2017-84890-P and by Junta de Andalucía group FQM-366.

multiplicity m and embedding dimension e are provided. We also compute the semigroups where these minimal values are achieved.

Keywords: Embedding dimension, Frobenius number, genus, multiplicity, numerical semigroup.

Math. Subj. Class.: 20M14, 20M05

Polgrupe s fiksno večkratnostjo in vložitveno dimenzijo*

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Prejeto 15. februarja 2019, sprejeto 11. septembra 2019, objavljeno na spletu 4. novembra 2019

Povzetek

Pri danem $m \in \mathbb{N}$ se številna polgrupa z večkratnostjo m imenuje pakirana številna polgrupa, če je njena minimalna množica generatorjev vsebovana v $\{m, m + 1, \dots, 2m - 1\}$. V tem članku so pakirane številne polgrupe uporabljene za izgradnjo množice številskih polgrup z dano večkratnostjo in vložitveno dimenzijo ter za tvorbo razčlenitve te množice. Wilfova domneva je potrjena v drevesu, pridruženem nekaterim pakiranim številskim polgrupam. Nadalje, če sta dani dve pozitivni celi števili m in e , je podanih nekaj algoritmov za izračun minimalnega Frobeniusovega števila in minimalnega rodu množice

* Avtorji se zahvaljujejo recenzentu in uredniku za njune koristne pripombe in predloge, ki so pomagali izboljšati ta članek.

† Delno podprt s strani MTM2017-84890-P in od Junta de Andalucía group FQM-366.

‡ Delno podprta s strani MTM2017-84890-P in od Junta de Andalucía group FQM-298.

§ Delno podprt s strani MTM2017-84890-P in od Junta de Andalucía group FQM-343.

¶ Delno podprt s strani MTM2015-65764-C3-1-P (MINECO/FEDER, UE), od MTM2017-84890-P in od Junta de Andalucía group FQM-366.

številskih polgrup z večkratnostjo m in vložitveno dimenzijo e . Izračunamo tudi polgrupe, pri katerih so te minimalne vrednosti dosežene.

Ključne besede: Vložitvena dimenzija, Frobeniusovo število, rod, večkratnost, številaska polgrupa.

Math. Subj. Class.: 20M14, 20M05
