

# Hypergeometric degenerate Bernoulli polynomials and numbers

Takao Komatsu \* 

*Department of Mathematical Sciences, School of Science,  
Zhejiang Sci-Tech University, Hangzhou 310018, China*

Received 11 January 2019, accepted 18 February 2020, published online 15 October 2020

---

## Abstract

Carlitz defined the degenerate Bernoulli polynomials  $\beta_n(\lambda, x)$  by means of the generating function  $t((1+\lambda t)^{1/\lambda} - 1)^{-1}(1+\lambda t)^{x/\lambda}$ . In 1875, Glaisher gave several interesting determinant expressions of numbers, including Bernoulli, Cauchy and Euler numbers. In this paper, we show some expressions and properties of hypergeometric degenerate Bernoulli polynomials  $\beta_{N,n}(\lambda, x)$  and numbers, in particular, in terms of determinants.

The coefficients of the polynomial  $\beta_n(\lambda, 0)$  were completely determined by Howard in 1996. We determine the coefficients of the polynomial  $\beta_{N,n}(\lambda, 0)$ . Hypergeometric Bernoulli numbers and hypergeometric Cauchy numbers appear in the coefficients.

*Keywords:* Bernoulli numbers, hypergeometric Bernoulli numbers, hypergeometric Cauchy numbers, hypergeometric functions, degenerate Bernoulli numbers, determinants, recurrence relations.

*Math. Subj. Class.:* 11B68, 11B37, 11C20, 15A15, 33C15

---

---

\*The author thanks the anonymous referee for careful reading of the manuscript and helpful comments and suggestions.

*E-mail address:* [komatsu@zstu.edu.cn](mailto:komatsu@zstu.edu.cn) (Takao Komatsu)

# Hipergeometrijski degenerirani Bernoullijevi polinomi in števila

Takao Komatsu \* 

*Department of Mathematical Sciences, School of Science,  
Zhejiang Sci-Tech University, Hangzhou 310018, China*

Prejeto 11. januarja 2019, sprejeto 18. februarja 2020, objavljeno na spletu 15. oktobra 2020

---

## Povzetek

Carlitz je definiral degenerirane Bernoullijeve polinome  $\beta_n(\lambda, x)$  s pomočjo rodovne funkcije  $t((1 + \lambda t)^{1/\lambda} - 1)^{-1}(1 + \lambda t)^{x/\lambda}$ . Leta 1875 je Glaisher predstavil zanimive determinantne izraze različnih števil, vključno z Bernoullijevimi, Cauchyjevimi in Eulerjevimi števili. V tem članku dokažemo nekaj izrazov in lastnosti hipergeometrijskih degeneriranih Bernoullijevih polinomov  $\beta_{N,n}(\lambda, x)$  in števil, še posebej takih, ki se izražajo s pomočjo determinant.

Koeficiente polinoma  $\beta_n(\lambda, 0)$  je v celoti določil Howard leta 1996. V tem članku določimo koeficiente polinoma  $\beta_{N,n}(\lambda, 0)$ . V teh koeficientih nastopajo hipergeometrijska Bernoullijeva števila in hipergeometrijska Cauchyjeva števila.

*Ključne besede: Bernoullijeva števila, hipergeometrijska Bernoullijeva števila, hipergeometrijska Cauchyjeva števila, hipergeometrijske funkcije, degenerirana Bernoullijeva števila, determinante, rekurzivne zveze.*

*Math. Subj. Class.: 11B68, 11B37, 11C20, 15A15, 33C15*

---

---

\* Avtor se zahvaljuje neznanemu recenzentu za skrbno branje rokopisa ter za koristne pripombe in predloge.  
*E-poštni naslov:* [komatsu@zstu.edu.cn](mailto:komatsu@zstu.edu.cn) (Takao Komatsu)