

Grundy domination and zero forcing in Kneser graphs*

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Abstract

In this paper, we continue the investigation of different types of (Grundy) dominating sequences. We consider four different types of Grundy domination numbers and the related zero forcing numbers, focusing on these numbers in the well-known class of Kneser graphs $K_{n,r}$. In particular, we establish that the Grundy total domination number $\gamma_{gr}^t(K_{n,r})$ equals $\binom{2r}{r}$ for any $r \geq 2$ and $n \geq 2r + 1$. For the Grundy domination number of Kneser graphs we get $\gamma_{gr}(K_{n,r}) = \alpha(K_{n,r})$ whenever n is sufficiently larger than r . On the other hand, the zero forcing number $Z(K_{n,r})$ is proved to be $\binom{n}{r} - \binom{2r}{r}$ when $n \geq 3r + 1$ and $r \geq 2$, while lower and upper bounds are provided for $Z(K_{n,r})$ when $2r + 1 \leq n \leq 3r$. Some lower bounds for different types of minimum ranks of Kneser graphs are also obtained along the way.

Keywords: Grundy domination number, Grundy total domination number, Kneser graph, zero forcing number, minimum rank.

Math. Subj. Class.: 05C69, 05C76, 05D05

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Grundyjeva dominacija in ničelna prisila v Kneserjevih grafih*

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Povzetek

V članku nadaljujemo z raziskavami različnih tipov (Grundyjevih) dominacijskih zaporedij. Obravnavamo štiri različne tipe Grundyjevih dominantnih števil in njim sorodna števila ničelne prisile, pri čemer se osredotočamo na ta števila v dobro znanih Kneserjevih grafih $K_{n,r}$. Med drugim ugotovimo, da je Grundyjevo celotno dominantno število $\gamma_{\text{gr}}^t(K_{n,r})$ enako $\binom{2r}{r}$ za vsaka $r \geq 2$ in $n \geq 2r + 1$. Za Grundyjevo dominantno število Kneserjevega grafa dobimo, da je $\gamma_{\text{gr}}(K_{n,r}) = \alpha(K_{n,r})$, če je le n dovolj veliko število v primerjavi z r . Dokažemo tudi, da je število ničelne prisile $Z(K_{n,r})$ enako $\binom{n}{r} - \binom{2r}{r}$, ko je $n \geq 3r + 1$ in $r \geq 2$, medtem ko za $Z(K_{n,r})$, ko je $2r + 1 \leq n \leq 3r$, najdemo spodnje in zgornje meje. Spotoma dobimo tudi nekaj spodnjih mej za različne tipe najmanjših rangov Kneserjevih grafov.

Ključne besede: Grundyjevo dominantno število, Grundyjevo celotno dominantno število, Kneserjev graf, število ničelne prisile, najmanjši rang.

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