


Mapification of n -dimensional abstract polytopes and hypertopes

Antonio Breda d’Azevedo * 

Department of Mathematics, University of Aveiro, Aveiro, Portugal

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Abstract

The n -dimensional abstract polytopes and hypertopes, particularly the regular ones, have gained great popularity over recent years. The main focus of research has been their symmetries and regularity. The planification of a polyhedron helps its spatial construction, yet it destroys symmetries. No “planification” of n -dimensional polytopes do exist, however it is possible to make a “mapification” of an n -dimensional polytope; in other words it is possible to construct a restrictedly-marked map representation of an abstract polytope on some surface that describes its combinatorial structures as well as all of its symmetries. There are infinitely many ways to do this, yet there is one that is more natural that describes reflections on the sides of $(n - 1)$ -simplices (flags or n -flags) with reflections on the sides of n -gons. The restrictedly-marked map representation of an abstract polytope is a cellular embedding of the flag graph of a polytope. We illustrate this construction with the 4-cube, a regular 4-polytope with automorphism group of size 384. This paper pays a tribute to Lynne James’ last work on map representations.

Keywords: Maps, hypermaps, representation, polytopes, hypertopes.

Math. Subj. Class.: 05C10, 05C25, 05C65, 05E18, 20F65

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E-mail address: breda@ua.pt (Antonio Breda d’Azevedo)

Ozemljevidenje n -dimenzionalnih abstraktnih politopov in hipertopov

Antonio Breda d’Azevedo * 

Department of Mathematics, University of Aveiro, Aveiro, Portugal

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Povzetek

V zadnjih nekaj letih so n -dimenzionalni abstraktni politopi in hipertopi, še posebej regularni, postali zelo priljubljeni. Glavna pozornost raziskav je bila namenjena njihovim simetrijam in regularnosti. Razgrnitev poliedra v ravnino sicer pomaga pri njegovi prostorski konstrukciji, vendar pa uniči simetrije. Čeprav ne obstaja nikakršno “oravninjenje” n -dimenzionalnih politopov, pa je mogoče nekakšno “ozemljevidenje” n -dimenzionalnega politopa; z drugimi besedami, mogoče je konstruirati omejeno označeno zemljevidno predstavitev abstraktnega politopa na neki ploskvi, ki opiše tako njegove kombinatorične strukture kot tudi vse njegove simetrije. To je mogoče storiti na neskončno mnogo načinov, vendar je eden, ki je bolj naraven kot drugi, in sicer v tem smislu, da opiše zrcaljenja na stranicah $(n - 1)$ -simpleksov (praporov ali n -praporov) s pomočjo zrcaljenj na stranicah n -kotnikov. Omejeno označena zemljevidna predstavitev abstraktnega politopa je celična vložitev grafa praporov tega politopa. To konstrukcijo ilustriramo s 4-kocko, regularnim 4-politopom z grupo avtmorfizmov velikosti 384. Ta članek je poklon Lynne Jamesovemu zadnjemu delu o predstavitvi zemljevidov.

Ključne besede: Zemljevidi, hiperzemljevidi, predstavitve, politopi, hipertopi.

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E-poštni naslov: breda@ua.pt (Antonio Breda d’Azevedo)