An equivalent formulation of the Fan-Raspaud Conjecture and related problems

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Abstract

In 1994, it was conjectured by Fan and Raspaud that every simple bridgeless cubic graph has three perfect matchings whose intersection is empty. In this paper we answer a question recently proposed by Mkrtchyan and Vardanyan, by giving an equivalent formulation of the Fan-Raspaud Conjecture. We also study a possibly weaker conjecture originally proposed by the first author, which states that in every simple bridgeless cubic graph there exist two perfect matchings such that the complement of their union is a bipartite graph. Here, we show that this conjecture can be equivalently stated using a variant of Petersen-colourings, we prove it for graphs having oddness at most four and we give a natural extension to bridgeless cubic multigraphs and to certain cubic graphs having bridges.

Keywords: Cubic graph, perfect matching, oddness, Fan-Raspaud Conjecture, Berge-Fulkerson Conjecture, Petersen-colouring.

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Enakovredna formulacija Fan-Raspaudove domneve in sorodni problemi

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Povzetek

Leta 1994 sta Fan in Raspaud postavila domnevo, da ima vsak enostavni brezmostni kubični graf tri popolna prirejanja, katerih presek je prazen. V tem članku odgovorimo na vprašanje, ki sta ga nedavno zastavila Mkrtchyan in Vardanyan, in sicer tako, da predstavimo ekvivalentno formulacijo Fan-Raspaudove domneve. Preučujemo tudi možno šibkejšo domnevo, ki jo je prvotno predstavil prvi avtor, češ da v vsakem enostavnem brezmostnem kubičnem grafu obstajata taki dve popolni prirejanji, da je komplement njune unije dvodelen graf. V tem članku pokažemo, da se da to domnevo enakovredno izraziti s pomočjo različice Petersenovih barvanj, dokažemo jo za grafe, katerih nenavadnost znaša največ štiri, predstavimo pa tudi njeno naravno razširitev na brezmostne kubične multigrafe in na določene kubične grafe, ki premorejo mostove.

Ključne besede: Kubični graf, popolno prirejanje, nenavadnost, Fan-Raspaudova domneva, Berge-Fulkersonova domneva, Petersenovo barvanje.

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