

Archimedean toroidal maps and their minimal almost regular covers*

Kostiantyn Drach †

Jacobs University Bremen, Bremen, Germany

Yurii Haidamaka

V. N. Karazin Kharkiv National University, Kharkiv, Ukraine

Mark Mixer

Wentworth Institute of Technology, Boston, United States of America

Maksym Skoryk

V. N. Karazin Kharkiv National University, Kharkiv, Ukraine

Received 20 October 2018, accepted 26 June 2019, published online 20 November 2019

Abstract

The automorphism group of a map acts naturally on its flags (triples of incident vertices, edges, and faces). An Archimedean map on the torus is called almost regular if it has as few flag orbits as possible for its type; for example, a map of type (4.8^2) is called almost regular if it has exactly three flag orbits. Given a map of a certain type, we will consider other more symmetric maps that cover it. In this paper, we prove that each Archimedean toroidal map has a unique minimal almost regular cover. By using the Gaussian and Eisenstein integers, along with previous results regarding equivelar maps on the torus, we construct these minimal almost regular covers explicitly.

Keywords: Maps, polytopes, groups, covers, Gaussian and Eisenstein integers.

Math. Subj. Class.: 52B15, 51M20, 52C22

*This project began during the Fields-MITACS 2011 Summer Undergraduate Research Program. We would like to thank the Fields Institute for its hospitality during that program. Additionally we thank Daniel Pellicer, Isabel Hubard, and Asia Weiss for their constant support, and for providing the question which has been solved in this paper. Also, we thank the referees for suggesting several improvements.

†The research of the first author was partially supported by the advanced grant 695 621 “HOLOGRAM” of the European Research Council (ERC), which is gratefully acknowledged.

E-mail addresses: k.drach@jacobs-university.de, kostya.drach@gmail.com (Kostiantyn Drach), galdamakay@gmail.com (Yurii Haidamaka), mixerm@wit.edu (Mark Mixer), skoryk.maksym@gmail.com (Maksym Skoryk)

Arhimedski torusni zemljevidi in njihovi minimalni skoraj regularni krovi*

Kostiantyn Drach †

Jacobs University Bremen, Bremen, Germany

Yurii Haidamaka

V. N. Karazin Kharkiv National University, Kharkiv, Ukraine

Mark Mixer

Wentworth Institute of Technology, Boston, United States of America

Maksym Skoryk

V. N. Karazin Kharkiv National University, Kharkiv, Ukraine

Prejeto 20. oktobra 2018, sprejeto 26. junija 2019, objavljeno na spletu 20. novembra 2019

Povzetek

Grupa avtomorfizmov zemljevida deluje naravno na njegovih praporih (trojicah incidentnih vozlišč, povezav in lic). Arhimedski zemljevid na torusu se imenuje skoraj regularen, če ima tako malo orbit praporov, kot je to le mogoče za njegov tip; na primer, zemljevid tipa (4.8^2) se imenuje skoraj regularen, če ima natanko tri orbite praporov. Za dani zemljevid danega tipa bomo obravnavali druge, bolj simetrične zemljevide, ki so njegovi krovi. V tem članku dokažemo, da ima vsak arhimedski torusni zemljevid en sam minimalen skoraj regularen krov. Z uporabo Gaussovih in Eisensteinovih celih števil, skupaj s predhodnimi rezultati, ki se nanašajo na ekvivalarne zemljevide na torusu, eksplicitno konstruiramo te minimalne skoraj regularne krove.

Ključne besede: Zemljevidi, politopi, grupe, krovi, Gaussova in Eisensteinova cela števila.

Math. Subj. Class.: 52B15, 51M20, 52C22

*Ta projekt se je začel med Fields-MITACS 2011 Summer Undergraduate Research Program. Radi bi se zahvalili Fieldsovem inštitutu za njegovo gostoljubje med tem programom. Radi bi se zahvalili tudi Danielu Pellicerju, Isabel Hubard, in Asii Weiss za njihovo stalno podporo, in za postavitev vprašanja, ki je bilo rešeno v tem članku. Prav tako se zahvaljujemo recenzentom za predlagane številne izboljšave.

†Raziskava prvega avtorja je bila delno podprta z napredno dotacijo 695 621 "HOLOGRAM" s strani European Research Council (ERC), kar s hvaležnostjo priznava.

E-poštni naslovi: k.drach@jacobs-university.de, kostya.drach@gmail.com (Kostiantyn Drach), galdamakay@gmail.com (Yurii Haidamaka), mixerm@wit.edu (Mark Mixer), skoryk.maksym@gmail.com (Maksym Skoryk)