

The existence of square non-integer Heffter arrays

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Abstract

A Heffter array $H(n; k)$ is an $n \times n$ matrix such that each row and column contains k filled cells, each row and column sum is divisible by $2nk + 1$ and either x or $-x$ appears in the array for each integer $1 \leq x \leq nk$. Heffter arrays are useful for embedding the graph K_{2nk+1} on an orientable surface. An integer Heffter array is one in which each row and column sum is 0. Necessary and sufficient conditions (on n and k) for the existence of an integer Heffter array $H(n; k)$ were verified by Archdeacon, Dinitz, Donovan and Yazıcı (2015) and Dinitz and Wanless (2017). In this paper we consider square Heffter arrays that are not necessarily integer. We show that such Heffter arrays exist whenever $3 \leq k < n$.

Keywords: Heffter arrays, biembedding cycle systems.

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Obstoj kvadratnih ne-celoštevilskih Heffterjevih matrik

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Povzetek

Heffterjeva matrika $H(n; k)$ je $n \times n$ matrika, v kateri vsaka vrstica in stolpec vsebuje k zapolnjenih mest, vsaka vrstična in stolpčna vsota je deljiva z $2nk + 1$ in bodisi x bodisi $-x$ nastopa v matriki za vsako celo število $1 \leq x \leq nk$. Heffterjeve matrike so koristne pri vložitvi grafa K_{2nk+1} na orientabilno ploskev. Celoštevilaska Heffterjeva matrika je takšna, pri kateri je vsota elementov vsake vrstice in stolpca enaka 0. Potrebne in zadostne pogoje (za n in k) za obstoj celoštevilske Heffterjeve matrike $H(n; k)$ so potrdili Archdeacon, Dinitz, Donovan in Yazıcı (2015) ter Dinitz in Wanless (2017). V tem članku obravnavamo kvadratne Heffterjeve matrike, ki niso nujno celoštevilske. Pokažemo, da takšne Heffterjeve matrike obstajajo vselej, kadar je $3 \leq k < n$.

Ključne besede: Heffterjeve matrike, bivložitveni ciklični sistemi.

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