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Reconfiguring vertex colourings of 2-trees

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Abstract

Let H be a graph and let $k \geq \chi(H)$ be an integer. The *k-colouring graph of H*, denoted $G_k(H)$, is the graph whose vertex set consists of all proper k-vertex-colourings (or simply k-colourings) of H using colours $\{1, 2, \ldots, k\}$; two vertices of $G_k(H)$ are adjacent if and only if the corresponding k-colourings differ in colour on exactly one vertex of H. If $G_k(H)$ has a Hamilton cycle, then H is said to have a *Gray code* of k-colourings, and the *Gray code number* of H is the least integer $k_0(H)$ such that $G_k(H)$ has a Gray code of k-colourings for all $k \geq k_0(H)$. Choo and MacGillivray determine the Gray code numbers of trees. We extend this result to 2-trees. A 2-*tree* is constructed recursively by starting with a complete graph on three vertices and connecting each new vertex to an existing clique on two vertices. We prove that if H is a 2-tree, then $k_0(H) = 4$ unless H is isomorphic to the join of a tree T and a vertex u, where T is a star on at least three vertices, or the bipartition of T has two even parts; in these cases, $k_0(H) = 5$.

Keywords: 2-trees, graph colouring, Gray codes, Hamilton cycles, reconfiguration problems.

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Rekonfiguracija vozliščnih barvanj 2-dreves

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Povzetek

Naj bo H graf in naj bo $k \geq \chi(H)$ celo število. *Graf k-barvanj grafa H*, označen z $G_k(H)$, je graf, katerega množica vozlišč sestoji iz vseh pravih k-vozliščnih barvanj (ali preprosto k-barvanj) grafa H z barvami $\{1, 2, \ldots, k\}$; vozlišči grafa $G_k(H)$ sta sosednji, če in samo če se ustrezni k-barvanji razlikujeta v barvi natanko enega vozlišča grafa H. Če ima $G_k(H)$ hamiltonski cikel, potem pravimo, da ima H *Grayevo kodo* iz k-barvanj. Najmanjše celo število $k_0(H)$, pri katerem ima $G_k(H)$ Grayevo kodo iz k-barvanj za vse $k \geq k_0(H)$, imenujemo *prag Grayeve kode* grafa H. Choo in MacGillivray sta določila pragove Grayevih kod za drevesa. V pričujočem članku razširimo ta rezultat na 2-drevesa. Konstrukcija 2-*drevesa* poteka rekurzivno: začnemo s polnim grafom na treh vozliščih, potem pa vsako novo vozlišče dodamo neki že obstoječi kliki na dveh vozliščih. Dokažemo, da če je H 2-drevo, potem je $k_0(H) = 4$, razen če je H izomorfen spoju drevesa T in vozlišča u, kjer je T zvezda na najmanj treh vozliščih, ali če se T deli na dva enako velika dela; v teh dveh primerih je $k_0(H) = 5$.

Ključne besede: 2-*drevesa, barvanje grafa, Grayeve kode, hamiltonski cikli, rekonfiguracijski problemi.*

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