

On identities of Watson type

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Abstract

We prove several identities of the type $\alpha(n) = \sum_{k=0}^{\infty} \beta\left(\frac{n-k(k+1)/2}{2}\right)$. Here, the functions $\alpha(n)$ and $\beta(n)$ count partitions with certain restrictions or the number of parts in certain partitions. Since Watson proved the identity for $\alpha(n) = Q(n)$, the number of partitions of n into distinct parts, and $\beta(n) = p(n)$, Euler's partition function, we refer to these identities as Watson type identities. Our work is motivated by results of G. E. Andrews and the second author who recently discovered and proved new Euler type identities. We provide analytic proofs and explain how one could construct bijective proofs of our results.

Keywords: Partitions, combinatorial identities, bijective combinatorics.

Math. Subj. Class.: 05C15, 05A17, 11P81, 11P84

O identitetah Watsonovega tipa

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Povzetek

Dokažemo več identitet tipa $\alpha(n) = \sum_{k=0}^{\infty} \beta\left(\frac{n-k(k+1)/2}{2}\right)$. Tukaj funkciji $\alpha(n)$ in $\beta(n)$ štejeta razčlenitve z določenimi omejitvami ali število delov v določenih razčlenitvah. Ker je Watson dokazal identiteto za $\alpha(n) = Q(n)$, kjer je $Q(n)$ število razčlenitev števila n na same različne dele, in za $\beta(n) = p(n)$, kjer je $p(n)$ Eulerjeva razčlenitvena funkcija, tovrstne identitete imenujemo identitete Watsonovega tipa. Najino delo je motivirano z rezultati G. E. Andrews in drugega avtorja, ki je nedavno odkril in dokazal nove identitete Eulerjevega tipa. Podava analitične dokaze in razloživa, kako konstruirati bijektivne dokaze najinih rezultatov.

Ključne besede: Razčlenitve, kombinatorične identitete, bijektivna kombinatorika.

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