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S_{12} and P_{12}-colorings of cubic graphs*

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Abstract

If G and H are two cubic graphs, then an H-coloring of G is a proper edge-coloring f with the edges of H, such that for each vertex x of G, there is a vertex y of H with $f(\partial_G(x)) = \partial_H(y)$. If G admits an H-coloring, then we will write $H \prec G$. The Petersen coloring conjecture of Jaeger (P_{10}-conjecture) states that for any bridgeless cubic graph G, one has: $P_{10} \prec G$. The S_{10}-conjecture states that for any cubic graph G, $S_{10} \prec G$. In this paper, we introduce two new conjectures that are related to these conjectures. The first of them states that any cubic graph with a perfect matching admits an S_{12}-coloring. The second one states that any cubic graph G whose edge-set can be covered with four perfect matchings, admits a P_{12}-coloring. We call these new conjectures S_{12}-conjecture and P_{12}-conjecture, respectively. Our first results justify the choice of graphs in S_{12}-conjecture and P_{12}-conjecture. Next, we characterize the edges of P_{12} that may be fictive in a P_{12}-coloring of a cubic graph G. Finally, we relate the new conjectures to the already known conjectures by proving that S_{12}-conjecture implies S_{10}-conjecture, and P_{12}-conjecture and $(5, 2)$-Cycle cover conjecture together imply P_{10}-conjecture. Our main tool for proving the latter statement is a new reformulation of $(5, 2)$-Cycle cover conjecture, which states that the edge-set of any claw-free bridgeless cubic graph can be covered with four perfect matchings.

Keywords: Cubic graph, Petersen graph, Petersen coloring conjecture, S_{10}-conjecture.

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S_{12} in P_{12}-barvanja kubičnih grafov*

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Povzetek

Če sta G in H kubična grafa, potem je H-barvanje grafa G pravilno povezavno barvanje f s povezavami grafa H, takšno da za vsako vozlišče x grafa G obstaja vozlišče y grafa H, za katero je $f(\partial_G(x)) = \partial_H(y)$. Če G dopušča H-barvanje, potem bomo pisali $H \prec G$. Jaegerjeva domneva o petersenskem barvanju (P_{10}-domneva) pravi, da za poljuben brezmostni kubični graf G velja $P_{10} \prec G$. S_{10}-domneva pravi, da za poljuben kubični graf G velja $S_{10} \prec G$. V tem članku vpeljeva dve novi domnevi, ki sta povezani s tema domnevama. Prva od njiju pravi, da poljuben kubični graf s popolnim prirejanjem dopušča S_{12}-barvanje. Druga pravi, da poljuben kubični graf G, katerega povezavno množico se da pokriti s štirimi popolnimi prirejanji, dopušča P_{12}-barvanje. Ti dve novi domnevi imenujeva S_{12}-domneva in P_{12}-domneva. Najin prvi rezultat opravičuje izbiro grafov v S_{12}-domnevi in P_{12}-domnevi. Nadalje, karakterizirava povezave v P_{12}, ki lahko nastopajo v P_{12}-barvanju kubičnega grafa G. Nazadnje, poveževa novi domnevi z že znanimi domnevami, ko dokaževa, da S_{12}-domneva implicira S_{10}-domnevo, in da P_{12}-domneva ter $(5, 2)$-ciklična krovna domneva skupaj implicirata P_{10}-domnevo. Najino glavno orodje za dokaz zadnje trditve je nova reformulacija $(5, 2)$-ciklične krovne domneve, ki pravi, da se povezavna množica poljubnega brezmostnega kubičnega grafa brez trizobov da pokriti s štirimi popolnimi prirejanji.

Ključne besede: Kubični graf, Petersenov graf, domneva o petersenskem barvanju, S_{10}-domneva.

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