

Ascending runs in permutations and valued Dyck paths*

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Abstract

We define a bijection between permutations and valued Dyck paths, namely, Dyck paths whose odd vertices are labelled with an integer that does not exceed their height. This map allows us to characterize the set of permutations avoiding the pattern 132 as the preimage of the set of Dyck paths with minimal labeling. Moreover, exploiting this bijection we associate to the set of n -permutations a polynomial that generalizes at the same time Eulerian polynomials, Motzkin numbers, super-Catalan numbers, little Schröder numbers, and other combinatorial sequences. Lastly, we determine the Hankel transform of the sequence of such polynomials.

Keywords: Permutation, Dyck path, pattern avoidance, Hankel transform.

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Naraščajoče lestvice v permutacijah in označenih Dyckovih poteh*

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Povzetek

Definiramo bijekcijo med permutacijami in označenimi Dyckovimi potmi, tj. Dyckovimi potmi, pri katerih je vsako liho vozlišče označeno s celim številom, ki ne presega njegove višine. S pomočjo te preslikave lahko karakteriziramo množico permutacij, ki ne vsebujejo vzorca 132, kot prasliko množice Dyckovih poti z minimalno označitvijo. To bijekcijo izkoristimo tudi za to, da množici n -permutacij priredimo polinom, ki hkrati posplošuje Eulerjeve polinome, Motzkinova števila, super-Catalanova števila, mala Schröderjeva števila in druga kombinatorična zaporedja. Določimo tudi Hankelovo transformiranko zaporedja takih polinomov.

Ključne besede: Permutacija, Dyckova pot, izogibanje vzorcem, Hankelova transformiranka.

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