

A q -queens problem VI. The bishops' period

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Abstract

The number of ways to place q nonattacking queens, bishops, or similar chess pieces on an $n \times n$ square chessboard is essentially a quasipolynomial function of n (by Part I of this series). The period of the quasipolynomial is difficult to settle. Here we prove that the empirically observed period 2 for three to ten bishops is the exact period for every number of bishops greater than 2. The proof depends on signed graphs and the Ehrhart theory of inside-out polytopes.

Keywords: Nonattacking chess pieces, Ehrhart theory, inside-out polytope, arrangement of hyperplanes, signed graph.

Math. Subj. Class.: 05A15, 00A08, 05C22, 52C07, 52C35

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Problem q kraljic VI. Perioda lovcev

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Povzetek

Število načinov, na katere lahko postavimo q nenapadajočih se kraljic, lovcev ali podobnih šahovskih figur na šahovnico $n \times n$, je v bistvu kvazipolinomska funkcija števila n (dokažemo v 1. članku iz te serije). Periodo kvazipolnoma je težko določiti. Dokažemo, da je empirično opažena perioda 2 za tri do deset lovcev natančna vrednost periode za vsako število lovcev, ki je večje od 2. Dokaz je odvisen od označenih grafov in Ehrhartove teorije izvihanih politopov.

Ključne besede: Nenapadajoče se šahovske figure, Ehrhartova teorija, izvihani politop, razporeditev hiperravnin, označeni graf.

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