

Also available at <http://amc-journal.eu>
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
Ars Mathematica Contemporanea Volume 5, Issue 1, Year 2012, Pages 107-126

Polyominoes with nearly convex columns: a semi-directed model

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Abstract

Column-convex polyominoes are by now a well-explored model. So far, however, no attention has been given to polyominoes whose columns can have either one or two connected components. This little known kind of polyominoes seems not to be manageable as a whole. To obtain solvable models, one needs to introduce some restrictions. This paper is focused on polyominoes with hexagonal cells. The restrictions just mentioned are semi-directedness and an upper bound (say m) on the size of the gap within a column. As the upper bound m grows, the solution of the model tends to break into more and more cases. We computed the area generating functions for $m = 1$, $m = 2$ and $m = 3$. In this paper, the $m = 1$ and $m = 2$ models are solved in full detail. To keep the size of the paper within reasonable limits, the result for the $m = 3$ model is stated without proof. The $m = 1$, $m = 2$ and $m = 3$ models have rational area generating functions, as column-convex polyominoes do. (It is practically sure, although we leave it unproved, that the area generating functions are also rational for $m = 4$, $m = 5$...)

However, the growth constants of the new models are 4.114908 and more, whereas the growth constant of column-convex polyominoes is 3.863131.

Keywords: Polyomino, hexagonal-celled, nearly convex column, semi-directed, area generating function.

Math Sci Net: [05B50](#)

Poliomine s skoraj konveksnimi stolpci: pol-usmerjen model

Povzetek

Stolpčno-konveksne poliomine so že dobro raziskane. Vendar doslej niso posvečali veliko pozornosti poliominam, katerih stolpci imajo bodisi eno bodisi dve povezani komponenti. Zdi se, da so te, slabo poznane poliomine, težko obvladljive v celoti. Da bi dobili rešljive modele, moramo uvesti določene omejitve. Ta članek se osredotoča na poliomine s šestkotniškimi celicami. Omejitve, ki smo jih omenili, pa se nanašajo na pol-usmerjenost in zgornjo mejo (recimo m) za vrzeli med stolpci. Ko zgornja meja m raste, se rešitev modela nagiba k temu, da se bo razvejala na več in več primerov. Izračunali smo rodovne funkcije ploščin za $m = 1$, $m = 2$ in $m = 3$. V tem članku so podrobno predstavljene rešitve modelov za $m = 1$ in $m = 2$. Da ne bi bil članek preobsežen, je rezultat modela za $m = 3$ naveden brez dokaza. Modeli za $m = 1$, $m = 2$ in $m = 3$ imajo racionalne rodovne funkcije ploščin, kot ga imajo tudi stolpčno-konveksne poliomine. (Čeprav tega nismo dokazali, je precej očitno, da so

rodovne funkcije ploščin racionalne tudi za $m = 4$, $m = 5 \dots$)
Vendar so konstante rasti za nove modele 4.114908 in več,
medtem ko je konstanta rasti za stolpčno-konveksne poliomine
3.863131.

Ključne besede: Poliomina, šestkotniške celice, skoraj
konveksen stolpec, pol-usmerjen, rodovne funkcije ploščin.