

4-edge-connected 4-regular maps on the projective plane*

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Abstract

In this paper rooted (near-)4-regular maps on the projective plane are investigated with respect to the root-valency, the number of edges, the number of inner faces, the number of nonroot-vertex-loops and the number of separating cycles. In particular, 4-edge connected 4-regular maps (which are related to the 3-flow conjecture by Tutte) are handled. Formulae of several types of rooted 4-edge-connected 4-regular maps on the projective plane are presented. Several known results on the number of 4-regular maps on the projective plane are also derived. Finally, using Darboux's method, a nice asymptotic formula for the numbers of this type of maps is given which implies that almost every (loopless) 4-regular map on the projective plane has a separating cycle.

Keywords: (Rooted) near-4-regular map, separating cycle, Lagrangian inversion, enumerating function, asymptotic.

Math. Subj. Class.: 05C10, 05C30, 05C45

*The computations of the approximate resultants were carried out with the aid of a symbolic algebra system (Maple [32]) on a Pentium II. The authors are grateful to the referees for their careful reading of this paper. Their constructive suggestion enabled us to make some major revisions which made this paper more readable and concrete. Supported by the the NNSFC under Grant No. 11171114, Science and Technology Commission of Shanghai Municipality (STCSM) under Grant No. 13dz2260400 and the Natural Science Foundation Project of CQ under Grant No. cstc2019jcyj-msxmX0724.

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4-povezavno-povezani 4-regularni zemljevidi na projektivni ravnini*

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Povzetek

V tem članku raziskujemo ukoreninjene (skoraj-)4-regularne zemljevide na projektivni ravnini glede na njihovo korenensko valenco, število povezav, število notranjih lic, število nekorenenskih vozliščnih zank in število ločevalnih ciklov. Še posebej obravnavamo 4-povezavno povezane 4-regularne zemljevide (ki so povezani s Tutteovo domnevo o 3-toku). Predstavimo formule za več vrst ukoreninjenih 4-povezavno povezanih 4-regularnih zemljevidov na projektivni ravnini. Izpeljemo tudi več znanih rezultatov glede števila 4-regularnih zemljevidov na projektivni ravnini. Nazadnje dobimo, s pomočjo Darbouxjeve metode, za število te vrste zemljevidov lepo asimptotsko formulo, iz katere sledi, da ima skoraj vsak (brezzančni) 4-regularen zemljevid na projektivni ravnini ločevalni cikel.

Ključne besede: (Ukoreninjen) skoraj-4-regularen zemljevid, ločevalni cikel, Lagrangeova inverzija, preštevalna funkcija, asimptotika.

Math. Subj. Class.: 05C10, 05C30, 05C45

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