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On the Hamilton-Waterloo problem: the case of two cycles sizes of different parity*

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Abstract

The Hamilton-Waterloo problem asks for a decomposition of the complete graph of order v into r copies of a 2-factor F_1 and s copies of a 2-factor F_2 such that $r+s = \left\lfloor \frac{v-1}{2} \right\rfloor$. If F_1 consists of m-cycles and F_2 consists of n cycles, we say that a solution to (m,n)-$\mathrm{HWP}(v;r,s)$ exists. The goal is to find a decomposition for every possible pair (r,s). In this paper, we show that for odd x and y, there is a solution to $(2^k x, y)$-$\mathrm{HWP}(vm;r,s)$ if $\gcd(x,y) \geq 3$, $m \geq 3$, and both x and y divide v, except possibly when $1 \in \{r,s\}$.

Keywords: 2-factorizations, Hamilton-Waterloo problem, Oberwolfach problem, cycle decomposition, resolvable decompositions.

Math. Subj. Class.: 05C51, 05C70

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O hamiltonsko-waterloojskem problemu: primer dveh ciklov, katerih velikosti imata različno parnost*

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Povzetek

Hamiltonsko-waterloojski problem sprašuje po dekompoziciji polnega grafa reda v na r kopij 2-faktorja F_1 in s kopij 2-faktorja F_2, pri čemer je $r + s = \lfloor \frac{v-1}{2} \rfloor$. Če F_1 sestoji iz m-ciklov in F_2 sestoji iz n ciklov, pravimo, da rešitev za (m, n)-$\mathrm{HWP}(v; r, s)$ obstaja. Cilj je najti dekompozicijo za vsak možen par (r, s). V tem članku pokažemo, da za liha x in y obstaja rešitev za $(2^k x, y)$-$\mathrm{HWP}(vm; r, s)$, če je $\gcd(x, y) \geq 3$, $m \geq 3$ in tako x kot y delita v, razen morda, ko je $1 \in \{r, s\}$.

Ključne besede: 2-faktorizacija, hamiltonsko-waterloojski problem, oberwolfaški problem, ciklična dekompozicija, rešljiva dekompozicija.

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