

The dimension of the negative cycle vectors of signed graphs

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Abstract

A *signed graph* is a graph Γ with edges labeled “+” and “−”. The sign of a cycle is the product of its edge signs. Let $\text{SpecC}(\Gamma)$ denote the list of lengths of cycles in Γ . We equip each signed graph with a vector whose entries are the numbers of negative k -cycles for $k \in \text{SpecC}(\Gamma)$. These vectors generate a subspace of $\mathbb{R}^{|\text{SpecC}(\Gamma)|}$. Using matchings with a strong permutability property, we provide lower bounds on the dimension of this space; in particular, we show for complete graphs, complete bipartite graphs, and a few other graphs that this space is all of $\mathbb{R}^{|\text{SpecC}(\Gamma)|}$.

Keywords: Signed graph, negative cycle vector, permutable matching.

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Dimenzija prostora vektorjev negativnih ciklov predznačenih grafov

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Povzetek

Predznačeni graf je graf Γ , katerega povezave so označene s “+” in “–”. Predznak cikla je produkt predznakov njegovih povezav. Naj $\text{SpecC}(\Gamma)$ označuje seznam dolžin ciklov v grafu Γ . Vsakemu predznačenemu grafu priredimo vektor, katerega komponente so števila negativnih k -ciklov za $k \in \text{SpecC}(\Gamma)$. Ti vektorji generirajo podprostor prostora $\mathbb{R}^{|\text{SpecC}(\Gamma)|}$. Z uporabo prirejanj s krepko permutabilnostno lastnostjo dobimo spodnje meje za dimenzijo tega prostora in pokažemo, da je za polne grafe, polne dvodelne grafe in še nekaj drugih grafov ta prostor enak celotnemu prostoru $\mathbb{R}^{|\text{SpecC}(\Gamma)|}$.

Ključne besede: Predznačeni graf, vektor negativnih ciklov, permutabilno prirejanje.

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