

Smallest snarks with oddness 4 and cyclic connectivity 4 have order 44*

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Abstract

The family of snarks – connected bridgeless cubic graphs that cannot be 3-edge-coloured – is well-known as a potential source of counterexamples to several important and long-standing conjectures in graph theory. These include the cycle double cover conjecture, Tutte’s 5-flow conjecture, Fulkerson’s conjecture, and several others. One way of approaching these conjectures is through the study of structural properties of snarks and construction of small examples with given properties. In this paper we deal with the problem of determining the smallest order of a nontrivial snark (that is, one which is cyclically 4-edge-connected and has girth at least 5) of oddness at least 4. Using a combination of structural analysis with extensive computations we prove that the smallest order of a snark with oddness at least 4 and cyclic connectivity 4 is 44. Formerly it was known that such a snark must have at least 38 vertices and one such snark on 44 vertices was constructed by Lukot’ka, Máčajová, Mazák and Škoviera in 2015. The proof requires determining all cyclically 4-edge-connected snarks on 36 vertices, which extends the previously compiled list of all such snarks up to 34 vertices. As a by-product, we use this new list to test the validity of several conjectures where snarks can be smallest counterexamples.

Keywords: Cubic graph, cyclic connectivity, edge-colouring, snark, oddness, computation.

Math. Subj. Class.: 05C15, 05C21, 05C30, 05C40, 05C75, 68R10

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Najmanjši snarki s čudnostjo 4 in ciklično povezanostjo 4 imajo red 44*

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Povzetek

Družina snarkov – povezanih brezmostnih kubičnih grafov, ki se jih ne da 3-povezavno pobarvati – je dobro znana kot možni vir protiprimerov za več pomembnih in dolgotrajnih domnev v teoriji grafov. Med njimi so domneva o dvojnem pokritju s cikli, Tuttejeva domneva o 5-toku, Fulkersonova domneva in več drugih. En način pristopa k tem domnevam je raziskovanje strukturnih lastnosti snarkov in konstruiranje majhnih primerov z danimi lastnostmi. V tem članku obravnavamo problem določitve najmanjšega reda netrivialnega snarka (takega, ki je ciklično 4-povezavno-povezan in ima ožino najmanj 5) in čudnost najmanj 4. S kombiniranjem strukturne analize in izčrpnih izračunov dokažemo, da je najmanjši red snarka s čudnostjo najmanj 4 in ciklično povezanostjo 4 enak 44. Prej je bilo znano, da mora imeti takšen snark najmanj 38 točk, in en tak snark na 44 točkah so konstruirali Lukot'ka, Máčajová, Mazák in Škoviera leta 2015. Dokaz zahteva določitev vseh cikličnih 4-povezavno-povezanih snarkov na 36 točkah, kar razširja predhodno zbrani seznam vseh takšnih snarkov z največ 34 točkami. Ta novi seznam uporabimo tudi za testiranje veljavnosti več domnev, kjer so snarki lahko najmanjši protiprimeri.

*Ključne besede: Kubičen graf, ciklična povezanost, povezavno barvanje, snark, čudnost, računanje.
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