ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 16 (2019) 49–58
https://doi.org/10.26493/1855-3974.1547.454
(Also available at http://amc-journal.eu)

On the parameters of intertwining codes*

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Received 8 December 2017, accepted 6 March 2018, published online 13 September 2018

Abstract

Let F be a field and let $F^{r\times s}$ denote the space of $r \times s$ matrices over F. Given equinumerous subsets $\mathcal{A} = \{A_i \mid i \in I\} \subseteq F^{r\times r}$ and $\mathcal{B} = \{B_i \mid i \in I\} \subseteq F^{s\times s}$ we call the subspace $C(\mathcal{A}, \mathcal{B}) := \{X \in F^{r\times s} \mid A_iX = XB_i \text{ for } i \in I\}$ an *intertwining code*. We show that if $C(\mathcal{A}, \mathcal{B}) \neq \{0\}$, then for each $i \in I$, the characteristic polynomials of A_i and B_i share a nontrivial factor. We give an exact formula for $k = \dim(C(\mathcal{A}, \mathcal{B}))$ and give upper and lower bounds. This generalizes previous work. Finally we construct intertwining codes with large minimum distance when the field is not 'too small'. We give examples of codes where $d = rs/k = 1/R$ is large where the minimum distance, dimension, and rate of the linear code $C(\mathcal{A}, \mathcal{B})$ are denoted by d, k, and $R = k/rs$, respectively.

Keywords: Linear code, dimension, distance.

Math. Subj. Class.: 94B65, 60C05

*The authors acknowledge the contribution of Robin Chapman who emailed us in August 2016 a proof of Theorem 2.8. His formula for $\dim(C(N_\lambda, N_\mu))$ involved a double sum which can be reduced to a single sum using Theorem 3.1. The authors gratefully acknowledge the support of the Australian Research Council Discovery Grant DP160102323.

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ISSN 1855-3966 (tiskana izd.), ISSN 1855-3974 (elektronska izd.)
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(Dostopno tudi na http://amc-journal.eu)

O parametrih prepletnih kod*

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Prejeto 8. decembra 2017, sprejeto 6. marca 2018, objavljeno na spletu 13. septembra 2018

Povzetek

Naj bo F obseg, $F^{r\times s}$ pa prostor $r \times s$ matrik nad F. Če sta podani enako močni podmnožici $\mathcal{A} = \{A_i \mid i \in I\} \subseteq F^{r\times r}$ in $\mathcal{B} = \{B_i \mid i \in I\} \subseteq F^{s\times s}$, potem pravimo podprostoru $C(\mathcal{A}, \mathcal{B}) := \{X \in F^{r\times s} \mid A_iX = XB_i \text{ za } i \in I\}$ *prepletna koda*. Pokažemo, da če je $C(\mathcal{A}, \mathcal{B}) \neq \{0\}$, potem si, za vsak $i \in I$, karakteristična polinoma matrik A_i in B_i delita netrivialen faktor. Izpeljemo natančno formulo za $k = \dim(C(\mathcal{A}, \mathcal{B}))$ in podamo zgornjo in spodnjo mejo. To je posplošitev prejšnjih rezultatov. Konstruiramo prepletne kode z veliko minimalno razdaljo v primerih, ko obseg ni ‘premajhen’. Podamo primere prepletov, kjer je $d = rs/k = 1/R$ velik, pri čemer so d, k, in $R = k/rs$ oznake za minimalno razdaljo, dimenzijo in velikost prepleta, v tem vrstnem redu.

Ključne besede: Linearna koda, dimenzija, razdalja.

Math. Subj. Class.: 94B65, 60C05

* Avtorja priznavata prispevek Robina Chapmana, ki nama je po e-pošti poslal avgusta 2016 dokaz Izreka 2.8. Njegova formula za $\dim(C(N_\lambda, N_\mu))$ je vsebovala dvojno vsoto, ki se z uporabo Izreka 3.1 prevede na enojno. Avtorja s hvaležnostjo priznavata podporo s strani Australian Research Council Discovery Grant DP160102323.

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