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On chromatic indices of finite affine spaces*

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Abstract

A line-coloring of the finite affine space $\mathrm{AG}(n,q)$ is *proper* if any two lines from the same color class have no point in common, and it is *complete* if for any two different colors i and j there exist two intersecting lines, one is colored by i and the other is colored by j. The pseudoachromatic index of $\mathrm{AG}(n,q)$, denoted by $\psi'(\mathrm{AG}(n,q))$, is the maximum number of colors in any complete line-coloring of $\mathrm{AG}(n,q)$. When the coloring is also proper, the maximum number of colors is called the achromatic index of $\mathrm{AG}(n,q)$. We prove that $\psi'(\mathrm{AG}(n,q)) \sim q^{1.5n-1}$ for even n, and that $q^{1.5(n-1)} < \psi'(\mathrm{AG}(n,q)) < q^{1.5n-1}$ for odd n. Moreover, we prove that the achromatic index of $\mathrm{AG}(n,q)$ is $q^{1.5n-1}$ for even n, and we provide the exact values of both indices in the planar case.

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Keywords: Achromatic index, complete coloring, finite affine space, pseudoachromatic index.
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O kromatičnih indeksih končnih afinih prostorov*

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Povzetek

Barvanje premic končnega afinega prostora $\mathrm{AG}(n,q)$ je *pravilno*, če nobeni dve premici iz istega barvnega razreda nimata nobene skupne točke, in *popolno*, če za poljubni dve različni barvi i in j obstajata dve sekajoči se premici, od katerih je ena pobarvana z i, druga pa z j. Psevdoakromatičnost prostora $\mathrm{AG}(n,q)$, označena s $\psi'(\mathrm{AG}(n,q))$, je maksimalno število barv v poljubnem barvanju premic prostora $\mathrm{AG}(n,q)$. Kadar je barvanje tudi pravilno, se maksimalno število barv imenuje akromatičnost prostora $\mathrm{AG}(n,q)$. Dokažemo, da je $\psi'(\mathrm{AG}(n,q)) \sim q^{1.5n-1}$ za sode n, in da je $q^{1.5(n-1)} < \psi'(\mathrm{AG}(n,q)) <$

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$q^{1.5n-1}$ za lihe n. Poleg tega, dokažemo da je akromatičnost prostora $\mathrm{AG}(n,q)$ enaka $q^{1.5n-1}$ za sode n, in podamo natančne vrednosti obeh indeksov v ravninskem primeru.

Ključne besede: Akromatičnost (ali akromatični indeks), popolno barvanje, končni afini prostor, psevdoakromatičnost.

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