

# A characterization of graphs with disjoint total dominating sets\*

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Received 8 November 2017, accepted 11 November 2018, published online 27 January 2019

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## Abstract

A set  $S$  of vertices in a graph  $G$  is a total dominating set of  $G$  if every vertex is adjacent to a vertex in  $S$ . A fundamental problem in total domination theory in graphs is to determine which graphs have two disjoint total dominating sets. In this paper, we solve this problem by providing a constructive characterization of the graphs that have two disjoint total dominating sets. Our characterization gives an entirely new description of graphs with two disjoint total dominating sets and places them in another context, developing them from four base graphs and applies a sequence of operations from seventeen operations that are independent and necessary to produce all such graphs. We show that every graph with two disjoint total dominating sets can be constructed using this method.

*Keywords:* Total domination number, disjoint total dominating sets.

*Math. Subj. Class.:* 05C69

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\*The authors express their sincere thanks to the referees for their meticulous and thorough reading of the paper, and for their very helpful comments which improved the exposition and clarity of the revised version of the paper. In particular, we thank one of the reviewers for suggesting to us Claim 4.9 and Claim 4.10 which greatly simplified the original proof.

†Research supported in part by the South African National Research Foundation and the University of Johannesburg.

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# Karakterizacija grafov z dvema disjunktnima totalno dominantnima množicama\*

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Prejeto 8. novembra 2017, sprejeto 11. novembra 2018, objavljeno na spletu 27. januarja 2019

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## Povzetek

Množica  $S$  točk v grafu  $G$  je totalno dominantna množica grafa  $G$ , če je vsaka točka grafa  $G$  sosedna točki iz  $S$ . Osnovni problem teorije totalne dominacije na grafih je določiti, kateri grafi imajo dve disjunktni totalno dominantni množici. V tem članku rešimo ta problem tako, da podamo konstruktivno karakterizacijo grafov, ki imajo dve disjunktni totalno dominantni množici. Naša karakterizacija daje popolnoma nov opis grafov z dvema disjunktnima dominantnima množicama in jih postavlja v drug kontekst, s tem da jih razvije iz štirih osnovnih grafov s pomočjo zaporedja operacij iz nabora sedemnajstih operacij, ki so neodvisne in potrebne za konstrukcijo vseh takšnih grafov. Pokažemo, da se da vsak graf z dvema disjunktnima totalno dominantnima množicama konstruirati z uporabo te metode.

*Ključne besede: Totalno dominantno število, disjunktni totalno dominantne množice.*

*Math. Subj. Class.: 05C69*

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\* Avtorja izražata svoje iskrene zahvale recenzentom za njihovo natančno in temeljito branje članka in za njihove zelo koristne pripombe, ki so izboljšale predstavitev in jasnost revidirane različice članka. Še posebej se zahvaljujema enemu od recenzentov, ki nama je predlagal trditev 4.9 in trditev 4.10, kar je zelo poenostavilo prvotni dokaz.

† Raziskavo sta delno podprla South African National Research Foundation in University of Johannesburg.

‡ Ta avtor je zaposlen tudi na Inštitutu za matematiko, fiziko in mehaniko, Jadranska 19, 1000 Ljubljana, Slovenija. Raziskavo je delno podprla Javna agencija za raziskovalno dejavnost Republike Slovenije s programom P1-0297 in projektom J1-9109.

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