ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 17 (2019) 141–151
https://doi.org/10.26493/1855-3974.1517.e42
(Also available at http://amc-journal.eu)

Graph characterization of fully indecomposable nonconvertible $(0, 1)$-matrices with minimal number of ones*

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Received 27 October 2017, accepted 15 July 2019, published online 10 September 2019

Abstract

Let A be a $(0, 1)$-matrix such that PA is indecomposable for every permutation matrix P and there are $2n + 3$ positive entries in A. Assume that A is also nonconvertible in a sense that no change of signs of matrix entries, satisfies the condition that the permanent of A equals to the determinant of the changed matrix.

We characterized all matrices with the above properties in terms of bipartite graphs. Here $2n + 3$ is known to be the smallest integer for which nonconvertible fully indecomposable matrices do exist. So, our result provides the complete characterization of extremal matrices in this class.

*The work of the second and the fourth authors was partially supported by Slovenian Research Agency (research core fundings No. P1-0288, No. P1-0222, and by grant BI-RU/16-18-033). The work of the first and the third authors is supported by Russian Scientific Foundation grant 17-11-01124.

The authors are especially thankful to the referee for communicated to them the gap which existed in Remark 3.15 of the original draft.

Keywords: Permanent, indecomposable matrices, graphs.
Math. Subj. Class.: 05C40, 15A27, 15A04, 05C50

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ARS MATHEMATICA CONTEMPORANEA

ISSN 1855-3966 (tiskana izd.), ISSN 1855-3974 (elektronska izd.)
ARS MATHEMATICA CONTEMPORANEA 17 (2019) 141–151
https://doi.org/10.26493/1855-3974.1517.e42
(Dostopno tudi na http://amc-journal.eu)

Grafovska karakterizacija popolnoma nerazcepnih nekonvertibilnih $(0, 1)$-matrik z minimalnim številom enic*

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Prejeto 27. oktobra 2017, sprejeto 12. julija 2019, objavljeno na spletu 10. septembra 2019

Povzetek

Bodi A matrika velikosti n-krat-n, ki ima $2n + 3$ elementov enakih 1, ostale pa 0. Denimo, da je PA nerazcepna za vsako permutacijsko matriko P in da A ni konvertibilna v smislu, da bi lahko njeno permanento, $\mathrm{per}(A)$, izračunali kot determinanto matrike, ki je dobljena iz A s spreminjanjem predznakov njenih elementov.

Opišemo vse matrike z zgornjimi lastnostmi s pomočjo dvodelnih grafov. Znano je, da mora imeti nekonvertibilna popolnoma nerazcepna $(0, 1)$-matrika vsaj $2n + 3$ enic. Naš rezultat torej popolnoma klasificira ekstremalne matrike znotraj tega razreda.

*Delo drugega in četrtega avtorja je bilo delno podprto s strani ARRS – Javne agencije za raziskovalno dejavnost Republike Slovenije (temeljna sredstva za raziskave št. P1-0288, št. P1-0222 in dotacija BI-RU/16-18-033). Delo prvega in tretjega avtorja je podprto s strani Russian Scientific Foundation, dotacija 17-11-01124.

Avtorji so posebej hvaležni recenzentu, da jih je opozoril na vrzel, ki je obstajala v opombi 3.15 prvotnega osnutka.

Ključne besede: Permanenta, nerazcepne matrike, grafi.
Math. Subj. Class.: 05C40, 15A27, 15A04, 05C50

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