

On graphs with exactly two positive eigenvalues*

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Abstract

The inertia of a graph G is defined to be the triplet $\text{In}(G) = (p(G), n(G), \eta(G))$, where $p(G)$, $n(G)$ and $\eta(G)$ are the numbers of positive, negative and zero eigenvalues (including multiplicities) of the adjacency matrix $A(G)$, respectively. Traditionally $p(G)$ (resp. $n(G)$) is called the positive (resp. negative) inertia index of G . In this paper, we introduce three types of congruent transformations for graphs that keep the positive inertia index and negative inertia index. By using these congruent transformations, we determine all graphs with exactly two positive eigenvalues and one zero eigenvalue.

Keywords: Congruent transformation, positive (negative) inertia index, nullity.

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O grafih z natančno dvema pozitivnima lastnima vrednostima*

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Povzetek

Vztrajnost grafa G je definirana kot urejena trojica $\text{In}(G) = (p(G), n(G), \eta(G))$, kjer so $p(G)$, $n(G)$ in $\eta(G)$ števila pozitivnih, negativnih in ničelnih lastnih vrednosti (upoštevaje njihove večkratnosti) matrice sosednosti $A(G)$. Tradicionalno se $p(G)$ (oz. $n(G)$) imenuje pozitivni (oz. negativni) vztrajnostni indeks grafa G . V tem članku vpeljemo tri vrste kongruentnih transformacij na grafih, ki ohranjajo tako pozitivni kot negativni vztrajnostni indeks. Z uporabo teh kongruentnih transformacij določimo vse grafe z natančno dvema pozitivnima in eno ničelno lastno vrednostjo.

Ključne besede: Kongruentna transformacija, pozitivni (negativni) vztrajnostni indeks, ničnost.

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