

# Direct product of automorphism groups of digraphs

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## Abstract

We study the direct product of automorphism groups of digraphs, where automorphism groups are considered as permutation groups acting on the sets of vertices. By a direct product of permutation groups  $(A, V) \times (B, W)$  we mean the group  $(A \times B, V \times W)$  acting on the Cartesian product of the respective sets of vertices. We show that, except for the infinite family of permutation groups  $S_n \times S_n$ ,  $n \geq 2$ , and four other permutation groups, namely  $D_4 \times S_2$ ,  $D_4 \times D_4$ ,  $S_4 \times S_2 \times S_2$ , and  $C_3 \times C_3$ , the direct product of automorphism groups of two digraphs is itself the automorphism group of a digraph. In the course of the proof, for each set of conditions on the groups  $A$  and  $B$  that we consider, we indicate or build a specific digraph product that, when applied to the digraphs representing  $A$  and  $B$ , yields a digraph whose automorphism group is the direct product of  $A$  and  $B$ .

*Keywords:* Digraph, automorphism group, permutation group, direct product.

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# Direktni produkt grup avtomorfizmov digrafov

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## Povzetek

Študiramo direktni produkt grup avtomorfizmov digrafov, pri čemer grupe avtomorfizmov obravnavamo kot permutacijske grupe, ki delujejo na množicah vozlišč. Direktni produkt permutacijskih grup  $(A, V) \times (B, W)$  je grupa  $(A \times B, V \times W)$ , ki deluje na kartezijanskem produktu ustreznih množic vozlišč. Pokažemo, da je direktni produkt grup avtomorfizmov dveh digrafov spet grupa avtomorfizmov digrafa razen v primeru neskončne družine permutacijskih grup  $S_n \times S_n$ ,  $n \geq 2$ , ter še štirih drugih permutacijskih grup in sicer  $D_4 \times S_2$ ,  $D_4 \times D_4$ ,  $S_4 \times S_2 \times S_2$  in  $C_3 \times C_3$ . To dokažemo tako, da za vsako obravnavano množico pogojev na grupah  $A$  in  $B$  navedemo ali zgradimo specifičen produkt digrafov. Če ta produkt uporabimo na digrafih, ki pripadata grupama  $A$  in  $B$ , dobimo digraf, katerega grupa avtomorfizmov je direktni produkt grup  $A$  in  $B$ .

*Ključne besede: Digraf, grupa avtomorfizmov, permutacijska grupa, direktni produkt.*

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