

# Distant sum distinguishing index of graphs with bounded minimum degree

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## Abstract

For any graph  $G = (V, E)$  with maximum degree  $\Delta$  and without isolated edges, and a positive integer  $r$ , by  $\chi'_{\Sigma, r}(G)$  we denote the  $r$ -distant sum distinguishing index of  $G$ . This is the least integer  $k$  for which a proper edge colouring  $c: E \rightarrow \{1, 2, \dots, k\}$  exists such that  $\sum_{e \ni u} c(e) \neq \sum_{e \ni v} c(e)$  for every pair of distinct vertices  $u, v$  at distance at most  $r$  in  $G$ . It was conjectured that  $\chi'_{\Sigma, r}(G) \leq (1 + o(1))\Delta^{r-1}$  for every  $r \geq 3$ . Thus far it has been in particular proved that  $\chi'_{\Sigma, r}(G) \leq 6\Delta^{r-1}$  if  $r \geq 4$ . Combining probabilistic and constructive approach, we show that this can be improved to  $\chi'_{\Sigma, r}(G) \leq (4 + o(1))\Delta^{r-1}$  if the minimum degree of  $G$  equals at least  $\ln^8 \Delta$ .

*Keywords: Distant sum distinguishing index of a graph, neighbour sum distinguishing index, adjacent strong chromatic index, distant set distinguishing index.*

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# Razlikovalni indeks oddaljenih vsot grafov z omejeno minimalno stopnjo

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## Povzetek

Za poljuben graf  $G = (V, E)$  z maksimalno stopnjo  $\Delta$  in brez izoliranih povezav ter za poljubno pozitivno celo število  $r$  označimo s  $\chi'_{\Sigma, r}(G)$  razlikovalni indeks  $r$ -oddaljenih vsot grafa  $G$ . To je najmanjše celo število  $k$ , za katerega obstaja takšno pravilno barvanje povezav  $c: E \rightarrow \{1, 2, \dots, k\}$ , da velja  $\sum_{e \ni u} c(e) \neq \sum_{e \ni v} c(e)$  za vsak par različnih vozlišč  $u, v$  na razdalji največ  $r$  v  $G$ . Postavljena je bila domneva, da velja  $\chi'_{\Sigma, r}(G) \leq (1 + o(1))\Delta^{r-1}$  za vsak  $r \geq 3$ . Doslej je bilo dokazano, da velja  $\chi'_{\Sigma, r}(G) \leq 6\Delta^{r-1}$ , če je  $r \geq 4$ . S povezovanjem verjetnostnega in konstruktivnega pristopa pokažemo, da lahko to izboljšamo na  $\chi'_{\Sigma, r}(G) \leq (4 + o(1))\Delta^{r-1}$ , če je minimalna stopnja grafa  $G$  enaka najmanj  $\ln^8 \Delta$ .

*Ključne besede:* Razlikovalni indeks oddaljenih vsot, razlikovalni indeks sosednjih vsot, sosedni krepki kromatični indeks, razlikovalni indeks oddaljenih množic.

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