

Extremal embedded graphs*

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Abstract

Let G be a ribbon graph and $\mu(G)$ be the number of components of the virtual link formed from G as a cellularly embedded graph via the medial construction. In this paper we first prove that $\mu(G) \leq f(G) + \gamma(G)$, where $f(G)$ and $\gamma(G)$ are the number of boundary components and Euler genus of G , respectively. A ribbon graph is said to be extremal if $\mu(G) = f(G) + \gamma(G)$. We then obtain that a ribbon graph is extremal if and only if its Petrial is plane. We introduce a notion of extremal minor and provide an excluded extremal minor characterization for extremal ribbon graphs. We also point out that a related result in the monograph by Ellis-Monaghan and Moffatt is not correct and prove that two related conjectures raised by Huggett and Tawfik hold for more general ribbon graphs.

Keywords: Ribbon graph, medial graph, Petrie dual, extremal minor, orientation.

Math. Subj. Class.: 05C10, 05C83, 57M15

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Ekstremalni vloženi grafi*

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Povzetek

Naj bo G trakovni graf in naj bo $\mu(G)$ število komponent virtualne povezave, tvorjene iz grafa G kot celično vložnega grafa preko medialne konstrukcije. V tem članku najprej dokažemo, da je $\mu(G) \leq f(G) + \gamma(G)$, kjer je $f(G)$ število robnih komponent, $\gamma(G)$ pa Eulerjev rod grafa G . Trakovni graf je ekstremalen, če je $\mu(G) = f(G) + \gamma(G)$. Nato pokažemo, da je trakovni graf ekstremalen, če in samo če je njegov Petrial ravninski graf. Vpeljemo pojem ekstremalnega minorja in podamo karakterizacijo ekstremalnih trakovnih grafov z izključenimi ekstremalnimi minorji. Izpostavimo tudi, da soroden rezultat v monografiji Ellis-Monaghana in Moffatta ni pravilen, ter dokažemo, da dve sorodni domnevi, ki sta ju postavila Huggett in Tawfik, veljata za splošnejše trakovne grafe.

Ključne besede: Trakovni graf, medialni graf, Petriejev dual, ekstremalni minor, orientacija.

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