

Also available at <http://amc-journal.eu>  
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)  
Ars Mathematica Contemporanea Volume 3, Issue 1, Year 2010, Pages 99-108

## **Reducibility of semigroups and nilpotent commutators with idempotents of rank two**

Matjaž Omladič, Heydar Radjavi

### **Abstract**

Let  $f$  be a noncommutative polynomial in two variables. Let  $S$  be a multiplicative semigroup of linear operators on a finite-dimensional vector space and  $T$  a fixed linear operator such that  $f(T, S)$  is nilpotent for all  $S$  in  $S$ . In [H. Radjavi, M. Omladič, *Nilpotent commutators and reducibility of semigroups*, Lin. and Multilin. Alg. 57 (2009), 307-317] the authors proposed questions, what one can say about the invariant subspace structure of  $S$  under this and other related conditions. In particular, they study the question under the condition that  $[S, T]^2 = 0$ , where  $T$  is a given idempotent of rank one. In this paper we extend some of the results given there to the case that  $T$  is a given idempotent of rank two.

**Keywords:** Reducibility, semigroups, commutators, nilpotent operators.

Math. Subj. Class.: 15A30, 47A15

Math Sci Net: [47A15 \(15A27 15A30\)](#)

## Reducibilnost polgrup in nilpotentnih komutatorjev z idempotenti ranga 2

### Povzetek

Naj bo  $f$  nekomutativni polinom dveh spremenljivk. Naj bo  $S$  multiplikativna polgrupa linearnih operatorjev na končno-razsežnem vektorskem prostoru in  $T$  tak linearni operator, da je  $f(T, S)$  nilpotent za vse  $S$  iz  $S$ . V [H. Radjavi, M. Omladič, *Nilpotent commutators and reducibility of semigroups*, Lin. and Multilin. Alg. 57 (2009), 307-317] avtorji postavijo vprašanje, kaj je mogoče povedati o invariantni podprostorski strukturi polgrupe  $S$  pod tem in ostalimi podobnimi pogoji. Še posebej raziščejo vprašanje pod pogojem  $[S, T]^2 = 0$ , kjer je  $T$  dani idempotent ranga ena. V tem članku razširimo nekaj od teh rezultatov na primer, ko je  $T$  dani idempotent ranga 2.

**Ključne besede:** Reducibilnost, polgrupe, komutatorji, nilpotentni operatorji.