

# On the generalized Oberwolfach problem

Andrea C. Burgess \*

*Department of Mathematics and Statistics, University of New Brunswick,  
100 Tucker Park Rd., Saint John, NB E2L 4L5, Canada*

Peter Danziger †

*Department of Mathematics, Ryerson University,  
350 Victoria St., Toronto, ON M5B 2K3, Canada*

Tommaso Traetta ‡

*DICATAM, University of Brescia, via Branze 43, 25123 Brescia, Italy*

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## Abstract

The generalized Oberwolfach problem  $OP_t(2w + 1; N_1, N_2, \dots, N_t; \alpha_1, \alpha_2, \dots, \alpha_t)$  asks for a factorization of  $K_{2w+1}$  into  $\alpha_i C_{N_i}$ -factors (where a  $C_{N_i}$ -factor of  $K_{2w+1}$  is a spanning subgraph whose components are cycles of length  $N_i \geq 3$ ) for  $i = 1, 2, \dots, t$ . Necessarily,  $N = \text{lcm}(N_1, N_2, \dots, N_t)$  is a divisor of  $2w + 1$  and  $w = \sum_{i=1}^t \alpha_i$ .

For  $t = 1$  we have the classic Oberwolfach problem. For  $t = 2$  this is the well-studied Hamilton-Waterloo problem, whereas for  $t \geq 3$  very little is known.

In this paper, we show, among other things, that the above necessary conditions are sufficient whenever  $2w + 1 \geq (t + 1)N$ ,  $\alpha_i > 1$  for every  $i \in \{1, 2, \dots, t\}$ , and  $\text{gcd}(N_1, N_2, \dots, N_t) > 1$ . We also provide sufficient conditions for the solvability of the generalized Oberwolfach problem over an arbitrary graph and, in particular, the complete equipartite graph.

*Keywords:* 2-factorizations, resolvable cycle decompositions, cycle systems, (generalized) Oberwolfach problem, Hamilton-Waterloo problem.

*Math. Subj. Class.:* 05C51, 05C70

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*E-mail addresses:* andrea.burgess@unb.ca (Andrea C. Burgess), danziger@ryerson.ca (Peter Danziger), tommaso.traetta@unibs.it (Tommaso Traetta)

# O posplošenem oberwolfaškem problemu

Andrea C. Burgess \*

*Department of Mathematics and Statistics, University of New Brunswick,  
100 Tucker Park Rd., Saint John, NB E2L 4L5, Canada*

Peter Danziger †

*Department of Mathematics, Ryerson University,  
350 Victoria St., Toronto, ON M5B 2K3, Canada*

Tommaso Traetta ‡

*DICATAM, University of Brescia, via Branze 43, 25123 Brescia, Italy*

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## Povzetek

Pri posplošenem oberwolfaškem problemu  $OP_t(2w + 1; N_1, N_2, \dots, N_t; \alpha_1, \alpha_2, \dots, \alpha_t)$  iščemo faktorizacijo grafa  $K_{2w+1}$  na  $\alpha_i C_{N_i}$ -faktorjev (kjer je  $C_{N_i}$ -faktor grafa  $K_{2w+1}$  vpeti podgraf, katerega komponente so cikli dolžine  $N_i \geq 3$ ) za  $i = 1, 2, \dots, t$ . Potrebni pogoj za rešitev je, da je  $N = \text{lcm}(N_1, N_2, \dots, N_t)$  delitelj števila  $2w + 1$  in da je  $w = \sum_{i=1}^t \alpha_i$ .

Za  $t = 1$  imamo klasični oberwolfaški problem. Za  $t = 2$  je to dobro raziskani hamilton-waterloojski problem, medtem ko je za  $t \geq 3$  znanega zelo malo.

V tem članku med drugim pokažemo, da je zgornji potrební pogoj tudi zadosten, kadar koli je  $2w + 1 \geq (t + 1)N$ ,  $\alpha_i > 1$  za vsak  $i \in \{1, 2, \dots, t\}$ , in  $\text{gcd}(N_1, N_2, \dots, N_t) > 1$ . Podamo tudi zadostne pogoje za rešljivost posplošenega oberwolfaškega problema nad poljubnim grafom in, še posebej, polnim ekvipartitnim grafom.

*Ključne besede: 2-faktorizacije, rešljive dekompozicije ciklov, sistemi ciklov, (posplošeni) oberwolfaški problem, hamilton-waterloojski problem.*

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*E-poštni naslovi:* andrea.burgess@unb.ca (Andrea C. Burgess), danziger@ryerson.ca (Peter Danziger), tommaso.traetta@unibs.it (Tommaso Traetta)