

# The validity of Tutte's 3-flow conjecture for some Cayley graphs\*

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## Abstract

Tutte's 3-flow conjecture claims that every bridgeless graph with no 3-edge-cut admits a nowhere-zero 3-flow. In this paper we verify the validity of Tutte's 3-flow conjecture on Cayley graphs of certain classes of finite groups. In particular, we show that every Cayley graph of valency at least 4 on a generalized dicyclic group has a nowhere-zero 3-flow. We also show that if  $G$  is a solvable group with a cyclic Sylow 2-subgroup and the connection sequence  $S$  with  $|S| \geq 4$  contains a central generator element, then the corresponding Cayley graph  $\text{Cay}(G, S)$  admits a nowhere-zero 3-flow.

*Keywords:* Nowhere-zero flow, Cayley graph, Tutte's 3-flow conjecture, connection sequence, solvable group, nilpotent group.

*Math. Subj. Class.:* 05C25, 05C21, 20D10

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## Veljavnost Tuttove domneve o 3-toku za nekatere Cayleyjeve grafe\*

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### Povzetek

Tuttova domneva o 3-toku trdi, da vsak graf, ki nima ne mostov ne 3-povezavnih rezov, dopušča povsod neničelni 3-tok. V tem članku potrdimo veljavnost Tuttove domneve o 3-toku za Cayleyjeve grafe določenih razredov končnih grup. Tako npr. pokažemo, da ima vsak Cayleyjev graf na posplošeni diciklični grupi valence 4 povsod neničeln 3-tok. Pokažemo tudi, da  $\text{Cay}(G, S)$  dopušča povsod neničelni 3-tok, če je  $G$  rešljiva grupa s ciklično Sylowsko 2-podgrupo.

*Ključne besede:* Povsod neničelni tok, Cayleyjev graf, Tuttova domneva o 3-toku, zaporedje povezav, rešljiva grupa, nilpotentna grupa.

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