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Pentavalent symmetric graphs of order four times an odd square-free integer*

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Abstract

A graph is said to be symmetric if its automorphism group is transitive on its arcs. Guo et al. in 2011 and Pan et al. in 2013 determined all pentavalent symmetric graphs of order $4pq$. In this paper, we shall generalize this result by determining all connected pentavalent symmetric graphs of order four times an odd square-free integer. It is shown in this paper that, for each such graph Γ, either the full automorphism group $\mathrm{Aut}\,\Gamma$ is isomorphic to $\mathrm{PSL}(2,p)$, $\mathrm{PGL}(2,p)$, $\mathrm{PSL}(2,p)\times\mathbb{Z}_2$ or $\mathrm{PGL}(2,p)\times\mathbb{Z}_2$, or Γ is isomorphic to one of 9 graphs.

Keywords: Arc-transitive graph, normal quotient, automorphism group.

Math. Subj. Class.: 05C25, 05E18, 20B25

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Petvalentni simetrični grafi, katerih red je štirikratnik lihega števila brez kvadratnih faktorjev*

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Povzetek

Za graf pravimo, da je simetričen, če je njegova grupa avtomorfizmov tranzitivna na množici njegovih lokov. Guo in dr. v 2011 ter Pan in dr. v 2013 so določili vse petvalentne simetrične grafe reda $4pq$. V tem članku posplošimo ta rezultat in določimo vse povezane petvalentne simetrične grafe, katerih red je štirikratnik lihega števila brez kvadratnih faktorjev. Pokažemo, da za vsak tak graf Γ velja, da je polna grupa avtomorfizmov $\mathrm{Aut}\,\Gamma$ izomorfna eni od grup $\mathrm{PSL}(2,p)$, $\mathrm{PGL}(2,p)$, $\mathrm{PSL}(2,p)\times\mathbb{Z}_2$ ali $\mathrm{PGL}(2,p)\times\mathbb{Z}_2$, ali pa je graf Γ izomorfen enemu od 9 grafov.

Ključne besede: Ločno-tranzitiven graf, normalni kvocient, grupa avtomorfizmov.

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