

Transversals in generalized Latin squares*

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Abstract

We are seeking a sufficient condition that forces a transversal in a generalized Latin square. A generalized Latin square of order n is equivalent to a proper edge-coloring of $K_{n,n}$. A transversal corresponds to a multicolored perfect matching. Akbari and Alipour defined $l(n)$ as the least integer such that every properly edge-colored $K_{n,n}$, which contains at least $l(n)$ different colors, admits a multicolored perfect matching. They conjectured that $l(n) \leq n^2/2$ if n is large enough. In this note we prove that $l(n)$ is bounded from above by $0.75n^2$ if $n > 1$. We point out a connection to anti-Ramsey problems. We propose a conjecture related to a well-known result by Woolbright and Fu, that every proper edge-coloring of K_{2n} admits a multicolored 1-factor.

Keywords: Latin squares, transversals, anti-Ramsey problems, Lovász local lemma.

Math. Subj. Class.: 05B15, 05C15, 60C05

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Transverzale v posplošenih latinskih kvadratih*

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Povzetek

Iščemo zadosten pogoj za obstoj transverzale v posplošenem latinskem kvadratu. Posplošeni latinski kvadrat reda n je ekvivalenten pravilnemu barvanju povezav grafa $K_{n,n}$. Transverzala ustreza večbarvnemu popolnemu prirejanju. Akbari in Alipour sta definirala $l(n)$ kot najmanjše celo število, za katero velja, da vsako pravilno barvanje povezav grafa $K_{n,n}$, ki vsebuje najmanj $l(n)$ različnih barv, dopušča večbarvno popolno prirejanje. Postavila sta domnevo, da je $l(n) \leq n^2/2$ če je n dovolj velik. V tem kratkem članku dokažemo, da je število $l(n)$ navzgor omejeno z $0.75n^2$ če je $n > 1$. Izpostavimo povezavo z anti-Ramseyevimi problemi. Postavimo domnevo, povezano z dobro znanim rezultatom Woolbrighta in Fuja, da vsako pravilno barvanje povezav grafa K_{2n} dopušča večbarvni 1-faktor.

Ključne besede: Latinski kvadrati, transverzale, anti-Ramseyjevi problemi, Lovászeva lokalna lema.

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