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Laudatio for Professor Dr. Wilfried Imrich on the occasion of his 75th birthday*

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Dear organizers, dear audience, and, of course, dear Wilfried!

Wilfried Imrich was born on May 25th, 1941 in Vienna and so we all have gathered here to express our best wishes on the occasion of his 75th birthday.

His childhood fell within the last years of World War II and the years following immediately after the war, which was no easy time for getting up regarding all aspects of life. In 1959 Wilfried began his studies of Mathematics and Physics at the University of Vienna. At this time no Bachelor or Master studies were known at Vienna University, but there was a so-called free scientific study in the classical Humboldtian sense, which directly led to a PhD degree, if after some years of attending lectures you were able to prepare a PhD thesis with new scientific results and to pass a so-called Rigorosum examination on the whole subject of Mathematics and a second discipline. No good system for students who were not gifted enough, but a very good system for students like Wilfried, to be initiated in doing mathematical research as early as possible.

Wilfried completed his studies with a thesis on Lattices and Volume in the area of the Geometry of Numbers in 1965 and started to work at the Technical University of Vienna, first as an assistant professor, later on achieving venia docendi in Mathematics and being appointed to associate professor in 1972. In the late sixties Wilfried held a position at New York State University at Albany. This was followed by a semester at Lomonossow University in Moscow and numerous visiting lecturer and researcher positions all over the world up to now. In 1973 he became full professor of Applied Mathematics at the University of Leoben, a position that he held until his retirement in 2009, thus being the professor serving for the third longest period in the history of this university. Wilfried advised several PhD theses in Mathematics and Engineering - some of his students are Professors of Mathematics now - and he was organizer and coorganizer of numerous national and international scientific conferences and meetings, in particular I would like to mention the Leoben-Ljubljana Graph Theory Seminars, that have become an institution in between.

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Wilfried headed the computer center of Leoben University for ten years and was responsible for the first so-called super-computer in Leoben. Of course, he also served for several academic positions like Dean of graduate studies or member of the advisory board of the Austrian Mathematical Society.

But let me switch to research now. In MathSciNet you will find 126 publications of Wilfried, mainly focusing on graph theory and algebraic combinatorics, including three books, with altogether about 1200 citations of his publications. There are almost 70 coauthors, and among them what could be called the nobility of graph theory. Because of his coauthorships with László Babai ([1], [4], [2], [3]), László Lovász ([4]) and Jaroslav Nešetřil ([23], [12]) Wilfried has Erdős number 2.

In the sixties there was a Privatdozent at Vienna University whose name was Herbert Izbicki, who gave courses and advised PhD theses in graph theory. Gerd Baron, who was one of Wilfried's colleagues at TU Vienna, had prepared a thesis in graph theory, too, and so Wilfried came in touch with the subject and started doing research on it.

Of course it is not possible to give here a detailed account of Wilfried's scientific achievements, so let me just mention a very personal choice of some of his results mainly in the area of graph products on a very short walk through the decades.

There are four main notions of graph products, usually named the Cartesian, the Direct, the Strong and the Lexicographic product, respectively. All four of them have as vertex set the Cartesian product of the vertex sets of the factors, but they differ in the definition of the edge sets, thereby offering discrete models for numerous applications. If $E(G_1)$ and $E(G_2)$ denote the edge set of the factors, then $((x_1, x_2), (y_1, y_2))$ is an edge of the product of G_1 and G_2 iff

- $((x_1, y_1) \in E(G_1)$ and $x_2 = y_2)$ or $(x_1 = y_1$ and $(x_2, y_2) \in E(G_2))$ (Cartesian product),
- $(x_1, y_1) \in E(G_1)$ and $(x_2, y_2) \in E(G_2)$ (Direct or Cardinal or Tensor or Categorical product),
- $((x_1, y_1) \in E(G_1)$ and $x_2 = y_2)$ or $(x_1 = y_1$ and $(x_2, y_2) \in E(G_2))$ or $((x_1, y_1) \in E(G_1)$ and $(x_2, y_2) \in E(G_2))$ (Strong product),
- $(x_1, y_1) \in E(G_1)$ or $(x_1 = y_1$ and $(x_2, y_2) \in E(G_2))$ (Lexicographic product).

Whereas the definitions in principle go back as far as to Whitehead and Russell for the first two products and to Hausdorff for the fourth kind of product (cf. [20], [30], [11]), a systematic study in the graph theoretic sense, which also includes the question of algorithmic recognition and factorization, became a research object in graph theory starting from the fifties and sixties of the last century. In particular I refer to the paper [29] by Gert Sabidussi (as most of you will know, also of Austrian origin, cf. [17]).

With respect to Wilfried's research there is already a publication in German from 1967 with the translated title "Cartesian product of set systems and graphs" [13] where he generalizes various results of Sabidussi on this type of product.

In 1969 Wilfried published in Archiv der Mathematik with the translated title "On the lexicographic product of graphs" [14]. In a 1959 paper Frank Harary [10] had asserted that two graphs commute in the lexicographic product if and only if they are both complete or isomorphic. It turned out that this assertion was incorrect and Wilfried could prove that they commute if and only if they are both complete or both of them are lexicographic powers of one graph H.

Let us jump into the seventies. I have to mention the 1975 paper by Wilfried and Herbert Izbicki entitled "Associative products of graphs" [18], where they could prove that exactly 20 of all possible kinds of products of graphs are associative (and almost all of them had been introduced by different authors). For tournaments it turns out that the only associative product is the lexicographic (and the antilexicographic) product.

Turning to the eighties and leaving the trace of product graphs for a moment I want to mention Wilfried's important 1984 paper in Combinatorica entitled "Explicit construction of regular graphs without small cycles" [15]. The girth of a graph is the length of a shortest cycle. Here Wilfried could prove that for every integer $d > 2$ one can effectively construct infinitely many Cayley graphs X of degree d whose girth $g(X)$ is larger than $0.4801 \cdot (\log n(X))/\log(d-1)) - 2$, where $n(X)$ is the number of vertices of X. For $d = 3$ even $g(X) > 0.9602 \cdot (\log n(X))/\log 2) - 5$ could be achieved. This improves a result by G.A.Margulis [26] and can be compared with a nonconstructive bound of Erdős and Sachs for regular graphs of degree d [6], which gives an asymptotic lower bound of $g(X) > \log(n(X))/\log(d-1) + 2$.

In a 1998 paper in Discrete Mathematics Wilfried deals with the algorithmic factorization of graphs with respect to the direct (or "cardinal") product [16]. Although the definition of the direct product is simple, it has some intricate properties. So the direct product may have high density, but small clique number or chromatic number, high independence number and high odd girth [20]. Every graph is an induced subgraph of a direct product of certain complete graphs [28], and the direct product has several interesting applications [20].

In 1985 Feigenbaum, Hershberger and Schäffer presented a polynomial time algorithm for factoring a graph with respect to the Cartesian product [7], and in 1992 Feigenbaum and Schäffer also presented an efficient algorithm for factorization with respect to the strong product [8]. From a paper of Miller [27] in 1968 it was known that a connected bipartite graph need not admit a unique prime factor decomposition with respect to the direct product. In his paper in question Wilfried was able to present a polynomial-time algorithm that yields prime factors of a finite, connected nonbipartite graph with respect to the direct product. The algorithm furthermore led to a new proof for the uniqueness if factoring a finite, connected nonbipartite graph with respect to the direct product.

I proceed to the 1998 paper of Wilfried in the European J. of Combinatorics together with Sandi Klavžar entitled "A convexity lemma and expansion procedures for bipartite graphs" [19]. In the paper the authors prove a characterization result for an induced connected subgraph H of a bipartite graph to be convex. This lemma is afterwards used to present a simple $\mathcal{O}(mn)$ algorithm for recognizing median graphs. A median graph is an undirected graph in which for every 3 vertices x_1, x_2, x_3 there exists a unique vertex, called the median, that belongs to shortest paths between each pair of x_1, x_2 and x_3. Median graphs go back as far as to Birkhoff and Kiss and have numerous applications (compare e.g. [5] or [25]). The authors also proposed the study of a hierarchy of graph classes starting from hypercubes over acyclic cubical complexes, median graphs, almost median graphs, semi-median graphs to partial cubes, where the aim was to achieve better recognition algorithms for median graphs and partial cubes (i.e., isometric subgraphs of hypercubes).

In a 1999 paper in SIAM J. Discrete Math. Wilfried, Sandi Klavžar and Henry Mulder could prove that the complexity of recognizing planar median graphs is linear [21].

Let us switch to our century and return to graph products.

Here we have to cite Wilfried's paper from 2007 with Iztok Peterin in Discrete Mathematics [24] where a recognition algorithm for a connected graph with respect to the Cartesian product is presented whose complexity is linear in time and space.

And, of course, I have to mention Wilfried's books. In his book on "Product graphs" together with Sandi Klavžar [20] that appeared 2000 in Wiley-Interscience and has more than 400 citations in MathSciNet the authors provide the algorithmic and structural issues relating the four most important notions of graph products.

In a second edition with Richard Hammack and Sandi Klavžar the "Handbook of product graphs" was published in 2011 [9] and provides now an extensive survey of the subject with up-to-date research results and conjectures.

In 2008 the book "Topics in graph theory. Graphs and their Cartesian products" coauthored by Wilfried, Sandi Klavžar and Douglas Rall was published [22]. Starting from basic facts, over classical topics, the study of graph invariants and notions of distance in graphs, it is finally demonstrated how the most important results on the structures and symmetries of Cartesian products lead to efficient factorization algorithms.

Let me end with some personal remarks:

My personal contacts to Wilfried go back to the 1979 meeting of the Austrian Mathematical Society in Leoben, and even more important to 1980, when Wilfried organized a meeting in Graph Theory and Combinatorics in Leoben, which gathered most people working in Austria within these areas. Without any doubt this meeting was one of the decisive points for me to undergo further research work in Combinatorics and Analysis of Algorithms in the forthcoming years.

Later on we met on several scientific conferences, he was so nice to invite me to his seminar in Leoben, and finitely our professional paths came close when I was appointed professor in Leoben in 1996. Wilfried always was to me a senior colleague and friend with a very young scientific attitude, broad knowledge and enthusiasm in his subject and far beyond and, as I am convinced, a kind of international reputation that only a very small number of contemporary Austrian mathematicians have been able to achieve.

Best wishes again, and *Ad multos annos*, to you, Wilfried, and thank you all.

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