

Order-chain polytopes*

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Abstract

Given two families X and Y of integral polytopes with nice combinatorial and algebraic properties, a natural way to generate a new class of polytopes is to take the intersection $\mathcal{P} = \mathcal{P}_1 \cap \mathcal{P}_2$, where $\mathcal{P}_1 \in X$, $\mathcal{P}_2 \in Y$. Two basic questions then arise: 1) when \mathcal{P} is integral and 2) whether \mathcal{P} inherits the “old type” from $\mathcal{P}_1, \mathcal{P}_2$ or has a “new type”, that is, whether \mathcal{P} is unimodularly equivalent to a polytope in $X \cup Y$ or not. In this paper, we focus on the families of order polytopes and chain polytopes. Following the above framework, we create a new class of polytopes which are named order-chain polytopes. When studying

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their volumes, we discover a natural relation with Ehrenborg and Mahajan's results on maximizing descent statistics.

Keywords: Poset, order-chain polytope, unimodular equivalence.

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Urejenostno-verižni politopi*

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Povzetek

Če sta dani dve družini X in Y celoštevilskih politopov z lepimi kombinatoričnimi in algebrajskimi lastnostmi, je en način za generiranje novega razreda politopov ta, da tvorimo presek $\mathcal{P} = \mathcal{P}_1 \cap \mathcal{P}_2$, kjer je $\mathcal{P}_1 \in X$, $\mathcal{P}_2 \in Y$. Tedaj se pojavita dve vprašanji: 1) kdaj je \mathcal{P} celoštevilski in 2) ali \mathcal{P} podeduje “stari tip” od $\mathcal{P}_1, \mathcal{P}_2$ ali pa ima “nov tip”, tj. ali je \mathcal{P} unimodularno ekvivalenten politopu v $X \cup Y$ ali ne. V tem članku se osredotočimo na družini urejenostnih politopov in verižnih politopov. Sledeč zgornjemu okvirju, ustvarimo nov razred politopov, urejenostno-verižne politope. Ko študiramo njihove prostornine, odkrijemo naravno zvezo z Ehrenborgovimi in Mahajanovimi rezultati v zvezi z maksimiziranjem statistike spusta.

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Ključne besede: Delno urejena množica, urejenostno-verižni politop, unimodularna ekvivalenca.

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