ARS MATHEMATICA CONTEMPORANEA

Also available at http://amc-journal.eu
ISSN 1855-3966 (printed ed.) ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 12 (2017) 383–413

Vertex-transitive graphs and their arc-types

Marston D. E. Conder*

Department of Mathematics, University of Auckland, Auckland, New Zealand

Tomaž Pisanski†

University of Primorska, Koper and IMFM, Ljubljana, Slovenia

Arjana Žitnik‡

University of Ljubljana and IMFM, Ljubljana, Slovenia

Abstract: Let X be a finite vertex-transitive graph of valency d, and let A be the full automorphism group of X. Then the *arc-type* of X is defined in terms of the sizes of the orbits of the stabiliser A_v of a given vertex v on the set of arcs incident with v. Such an orbit is said to be *self-paired* if it is contained in an orbit Δ of A on the set of all arcs of X such that Δ is closed under arc-reversal. The arc-type of X is then the partition of d as the sum $n_1+n_2+\cdots+n_t+(m_1+m_1)+(m_2+m_2)+\cdots+(m_s+m_s)$, where $n_1, n_2, \ldots, n_t$ are the sizes of the self-paired orbits, and $m_1, m_1, m_2, m_2, \ldots, m_s, m_s$ are the sizes of the non-self-paired orbits, in descending order. In this paper, we find the arc-types of several families of graphs. Also we show that the arc-type of a Cartesian product of two 'relatively prime' graphs is the natural sum of their arc-types. Then using these observations, we show that with the exception of $1+1$ and $(1+1)$, every partition as defined above is *realisable*, in the sense that there exists at least one vertex-transitive graph with the given partition as its arc-type.

Keywords: Symmetry type, vertex-transitive graph, arc-transitive graph, Cayley graph, Cartesian product, covering graph.

Math. Subj. Class.: 05E18, 20B25, 05C75, 05C76

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*This work was supported in part by the N.Z. Marsden Fund (via grant UOA1323).

E-mail addresses: m.conder@auckland.ac.nz (Marston D. E. Conder), tomaz.pisanski@upr.si (Tomaž Pisanski), arjana.zitnik@fmf.uni-lj.si (Arjana Žitnik)

†Research supported in part by the ARRS (via grants P1–0294, N1-0032, L7-5554, J1-7051, J1-6720.

‡Research supported in part by the ARRS (via grant P1–0294) and the European Science Foundation (Eurocores Eurogiga, GReGAS (N1–0011)).

ARS MATHEMATICA CONTEMPORANEA

Dostopno tudi na http://amc-journal.eu
ISSN 1855-3966 (tiskana izd.) ISSN 1855-3974 (elektronska izd.)
ARS MATHEMATICA CONTEMPORANEA 12 (2017) 383–413

Vozliščno-tranzitivni grafi in njihovi ločni tipi

Marston D. E. Conder*
Department of Mathematics, University of Auckland, Auckland, New Zealand

Tomaž Pisanski†
Univerza na Primorskem, Koper in IMFM, Ljubljana, Slovenia

Arjana Žitnik‡
Univerza v Ljubljani in IMFM, Ljubljana, Slovenia

Povzetek: Naj bo X končen vozliščno-tranzitiven graf valence d, in naj bo A polna grupa avtomorfizmov grafa X. Potem je *ločni tip* grafa X definiran v smislu velikosti orbit stabilizatorja A_v danega vozlišča v na množici lokov incidentnih z v. Za takšno orbito pravimo, da je sebi-prirejena, če je vsebovana v orbiti Δ grupe A na množici vseh takšnih lokov grafa X, za katere je orbita Δ zaprta za obračanje lokov. Ločni tip grafa X je tedaj razčlenitev števila d v vsoto $n_1 + n_2 + \cdots + n_t + (m_1 + m_1) + (m_2 + m_2) + \cdots + (m_s + m_s)$, kjer so $n_1, n_2, \ldots, n_t$ velikosti sebi-prirejenih orbit, $m_1, m_1, m_2, m_2, \ldots, m_s, m_s$ pa so velikosti sebi-neprirejenih orbit, v padajočem redu. V tem članku najdemo ločne tipe več družin grafov. Pokažemo tudi, da je ločni tip kartezičnega produkta dveh 'tujih si' grafov naravna vsota njunih ločnih tipov. Potem na podlagi teh opažanj pokažemo, da je, z izjemo $1+1$ in $(1+1)$, vsaka particija, kot je definirana zgoraj, *realizabilna*, v tem smislu, da obstaja vsaj en vozliščno-tranzitiven graf z dano razčlenitvijo kot svojim ločnim tipom.

Ključne besede: Simetrijski tip, vozliščno-tranzitiven graf, ločno-tranzitiven graf, Cayleyev graf, kartezični produkt, krovni graf.

Math. Subj. Class.: 05E18, 20B25, 05C75, 05C76

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*To delo je delno podprl N.Z. Marsden Fund (s sredstvi UOA1323).
E-poštni naslovi: m.conder@auckland.ac.nz (Marston D. E. Conder), tomaz.pisanski@upr.si (Tomaž Pisanski), arjana.zitnik@fmf.uni-lj.si (Arjana Žitnik)
†Raziskavo je delno podprla ARRS (s sredstvi P1–0294, N1-0032, L7-5554, J1-7051, J1-6720.
‡Raziskavo je delno podprla ARRS (s sredstvi P1–0294) in European Science Foundation (Eurocores Eurogiga, GReGAS (N1–0011)).