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## A normal quotient analysis for some families of oriented four-valent graphs\*

Jehan A. Al-bar, Ahmad N. Al-kenani, Najat Mohammad Muthana<sup>†</sup>

*King Abdulaziz University, Jeddah, Saudi Arabia*

Cheryl E. Praeger<sup>‡</sup>

*The University of Western Australia, Crawley, Australia; also affiliated with King Abdulaziz University, Jeddah, Saudi Arabia*

**Abstract:** We analyse the normal quotient structure of several infinite families of finite connected edge-transitive, four-valent oriented graphs. These families were singled out by Marušič and others to illustrate various different internal structures for these graphs in terms of their alternating cycles (cycles in which consecutive edges have opposite orientations). Studying the normal quotients gives fresh insights into these oriented graphs: in particular we discovered some unexpected ‘cross-overs’ between these graph families when we formed normal quotients. We determine which of these oriented graphs are ‘basic’, in the sense that their only proper normal quotients are degenerate. Moreover, we show that the three types of edge-orientations studied are the only orientations, of the underlying undirected graphs in these families, which are invariant under a group action which is both vertex-transitive and edge-transitive.

**Keywords:** Edge-transitive graph, oriented graph, cyclic quotient graph, transitive group.

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<sup>†</sup>*E-mail addresses:* jalbar@kau.edu.sa (Jehan A. Al-bar), analkenani@kau.edu.sa (Ahmad N. Al-kenani), nmuthana@kau.edu.sa (Najat Mohammad Muthana), cheryl.praeger@uwa.edu.au (Cheryl E. Praeger)

<sup>‡</sup>corresponding author

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## Normalna kvocientna analiza za nekatere družine orientiranih štirivalentnih grafov\*

Jehan A. Al-bar, Ahmad N. Al-kenani, Najat Mohammad Muthana<sup>†</sup>

*King Abdulaziz University, Jeddah, Saudi Arabia*

Cheryl E. Praeger<sup>‡</sup>

*The University of Western Australia, Crawley, Australia; zaposlena tudi na King Abdulaziz University, Jeddah, Saudi Arabia*

**Povzetek:** Analiziramo normalno kvocientno strukturo več neskončnih družin končnih, povezavno-tranzitivnih, štirivalentnih orientiranih grafov. Te družine so prvi preučevali Marušič in drugi, da so ilustrirali različne notranje strukture teh grafov v smislu njihovih alternirajočih ciklov (tj. ciklov, v katerih imajo zaporedne povezave nasprotno orientacije). Raziskovanje normalnih kvocientov daje svež vpogled v te orientirane grafe: ko smo tvorili normalne kvociente, smo odkrili tudi nekaj nepričakovanih ‘prekrivanj’ med temi družinami grafov. Določimo, kateri od teh orientiranih grafov so ‘osnovni’, v smislu, da so njihovi edini pravi normalni kvocienti degenerirani. Še več, pokažemo da so študirani trije tipi orientacij povezav edine orientacije osnovnih neusmerjenih grafov v teh družinah, ki so invariantne na delovanje grupe, ki je tako vozliščno-tranzitivno kot tudi povezavno-tranzitivno.

**Ključne besede:** Povezavno-tranzitivni graf, orientirani graf, ciklični kvocientni graf, tranzitivna grupa.

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<sup>†</sup>*E-poštni naslovi:* jalbar@kau.edu.sa (Jehan A. Al-bar), analkenani@kau.edu.sa (Ahmad N. Al-kenani), nmuthana@kau.edu.sa (Najat Mohammad Muthana), cheryl.praeger@uwa.edu.au (Cheryl E. Praeger)

<sup>‡</sup>kontaktni avtor