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## Spherical quadrangles with three equal sides and rational angles

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**Abstract:** When the condition of having three equal sides is imposed upon a (convex) spherical quadrangle, the four angles of that quadrangle cannot longer be freely chosen but must satisfy an identity. We derive two simple identities of this kind, one involving ratios of sines, and one involving ratios of tangents, and improve upon an earlier identity by Ueno and Agaoka.

The simple form of these identities enable us to further investigate the case in which all of the angles are rational multiples of  $\pi$  and produce a full classification, consisting of 7 infinite classes and 29 sporadic examples. Apart from being interesting in its own right, these quadrangles play an important role in the study of spherical tilings by congruent quadrangles.

**Keywords:** Spherical quadrangle, rational angle, spherical tiling.

Math. Subj. Class.: 51M09, 52C20, 11Y50

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## Sferični četverkotniki s tremi enakimi stranicami in racionalnimi koti

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**Povzetek:** Kadar (konveksen) sferičen četverkotnik izpolnjuje pogoj, da ima tri enake stranice, tedaj štirih kotov tega četverkotnika ni več mogoče svobodno izbrati, ampak morajo zadoščati neki identiteti. Izpeljemo dve enostavni identiteti te vrste, eno z razmerji sinusov, drugo z razmerji tangent, in izboljšamo identiteto, ki sta jo odkrila Ueno in Agaoka.

Enostavna oblika teh identitet nam omogoča nadaljnje raziskovanje primera, v katerem so vsi koti racionalni večkratniki od  $\pi$  in tako dobimo izčrpno klasifikacijo, ki sestoji iz 7 neskončnih družin in 29 dodatnih primerov. Poleg tega da so zanimivi sami po sebi, ti četverkotniki igrajo pomembno vlogo pri raziskovanju sferičnih tlakovanj s skladnimi četverkotniki.

**Ključne besede:** Sferični četverkotniki, racionalni koti, sferično tlakovanje.

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