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The existence of square integer Heffter arrays

Jeffrey H. Dinitz*

*Department of Mathematics and Statistics,
University of Vermont, Burlington, VT 05405, USA*

Ian M. Wanless†

School of Mathematical Sciences, Monash University, Vic 3800, Australia

Abstract: An integer Heffter array $H(m, n; s, t)$ is an $m \times n$ partially filled matrix with entries from the set $\{\pm 1, \pm 2, \dots, \pm ms\}$ such that *i*) each row contains s filled cells and each column contains t filled cells, *ii*) every row and column sums to 0 (in \mathbb{Z}), and *iii*) no two entries agree in absolute value. Heffter arrays are useful for embedding the complete graph K_{2ms+1} on an orientable surface in such a way that each edge lies between a face bounded by an s -cycle and a face bounded by a t -cycle. In 2015, Archdeacon, Dinitz, Donovan and Yazıcı constructed square (i.e. $m = n$) integer Heffter arrays for many congruence classes. In this paper we construct square integer Heffter arrays for all the cases not found in that paper, completely solving the existence problem for square integer Heffter arrays.

Keywords: Heffter array, biembedding.

Math. Subj. Class.: 05B20, 05C10

*E-mail addresses: jeff.dinitz@uvm.edu (Jeffrey H. Dinitz), ian.wanless@monash.edu (Ian M. Wanless).

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Obstoj kvadratnih celoštevilskih Heffterjevih matrik

Jeffrey H. Dinitz*

*Department of Mathematics and Statistics,
University of Vermont, Burlington, VT 05405, USA*

Ian M. Wanless†

School of Mathematical Sciences, Monash University, Vic 3800, Australia

Povzetek: Celoštevilska Heffterjeva matrika $H(m, n; s, t)$ je $m \times n$ delno zapolnjena matrika z elementi iz množice $\{\pm 1, \pm 2, \dots, \pm ms\}$ tako da *i*) vsaka vrstica vsebuje s zapolnjenih polj in vsak stolpec vsebuje t zapolnjenih polj, *ii*) vsota elementov v vsaki vrstici in vsakem stolpcu je enaka nič (v \mathbb{Z}), in *iii*) nobena dva elementa nimata enake absolutne vrednosti. Heffterjeve matrike so koristne pri vlaganju polnega grafa K_{2ms+1} na orientabilno ploskev tako da vsaka povezava leži med nekim licem omejenim z s -ciklom in nekim licem omejenim s t -ciklom. Leta 2015 so Archdeacon, Dinitz, Donovan in Yazıcı konstruirali kvadratne (tj. $m = n$) celoštevilске Heffterjeve matrike za mnoge kongruenčne razrede. V tem članku konstruiramo kvadratne celoštevilске Heffterjeve matrike za vse primere, ki jih Archdeacon, Dinitz, Donovan in Yazıcı v svojem članku niso obravnavali, in popolnoma rešimo problem obstoja za kvadratne celoštevilске Heffterjeve matrike.

Ključne besede: Heffterjeva matrika, dvojna vložitev.

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**E-poštni naslovi:* jeff.dinitz@uvm.edu (Jeffrey H. Dinitz), ian.wanless@monash.edu (Ian M. Wanless).

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