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## Immersing small complete graphs

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### Abstract

Following in the spirit of the Hadwiger and Hajós conjectures, Abu-Khzam and Langston have conjectured that every  $k$ -chromatic graph contains an immersion of  $K_k$ . They proved this for  $k \leq 4$ . Much before that, Lescure and Meyniel [F. Lescure and H. Meyniel, On a problem upon configurations contained in graphs with given chromatic number, Graph theory in memory of G. A. Dirac (Sandbjerg, 1985), 325–331, Ann. Discrete Math. 41, North-Holland, Amsterdam, 1989] obtained a stronger result that included also the values  $k = 5$  and 6, by proving that every simple graph of minimum degree  $k - 1$  contains an immersion of  $K_k$ . They noted that they also have a proof of the same result for  $k = 7$  but have not published it due to the length of the proof. We give a simple proof of this result. This, in particular, proves the conjecture of Abu-Khzam and Langston for every  $k \leq 7$ .

**Keywords:** Immersion, Hadwiger Conjecture.

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## Vložitve majhnih polnih grafov

### Povzetek

V duhu domnev Hadwigerja in Hajósa sta Abu-Khizam in Langston postavila domnevo, da vsak  $k$ -kromatski graf vsebuje podgraf  $K_k$ . To sta pokazala za  $k \leq 4$ . Precej pred tem sta Lescure in Meyniel [F. Lescure and H. Meyniel, On a problem upon configurations contained in graphs with given chromatic number, Graph theory in memory of G. A. Dirac (Sandbjerg, 1985), 325–331, Ann. Discrete Math. 41, North-Holland, Amsterdam, 1989] dokazala močnejši rezultat, ki je vseboval tudi primera  $k = 5$  in  $k = 6$ , in sicer da vsak enostaven graf minimalne stopnje  $k - 1$  vsebuje graf  $K_k$ . Omenila sta, da imata dokaz enakega rezultata za  $k = 7$ , ki pa ga zaradi dolžine dokaza nista objavila. V tem članku podamo enostaven dokaz tega rezultata. To tedaj dokaže tudi domnevo Abu-Khizam-a in Langston-a za vsak  $k \leq 7$ .

**Ključne besede:** Vložitev, Hadwigerjeva domneva.

