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Classification of regular balanced Cayley maps of minimal non-abelian metacyclic groups*

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Abstract: In this paper, we classify the regular balanced Cayley maps of minimal non-abelian metacyclic groups. Besides the quaternion group Q_8, there are two infinite families of such groups which are denoted by $M_{p,q}(m,r)$ and $M_p(n,m)$, respectively. Firstly, we prove that there are regular balanced Cayley maps of $M_{p,q}(m,r)$ if and only if $q=2$ and we list all of them up to isomorphism. Secondly, we prove that there are regular balanced Cayley maps of $M_p(n,m)$ if and only if $p=2$ and $n=m$ or $n=m+1$ and there is exactly one such map up to isomorphism in either case. Finally, as a corollary, we prove that any metacyclic p-group for odd prime number p does not have regular balanced Cayley maps.

Keywords: Regular balanced Cayley map, minimal non-abelian group, metacyclic group.

Math. Subj. Class.: 05C25, 05C30

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Klasifikacija regularnih uravnoteženih Cayleyevih zemljevidov minimalnih neabelskih metacikličnih grup*

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Povzetek: V tem članku klasificiramo regularne uravnotežene Cayleyeve zemljevide minimalnih neabelskih metacikličnih grup. Poleg kvaternionske grupe Q_8 sta še dve neskončni družini takšnih grup, ki jih označujemo z $M_{p,q}(m,r)$ in $M_p(n,m)$. Najprej pokažemo, da obstajajo uravnoteženi Cayleyevi zemljevidi grupe $M_{p,q}(m,r)$ natanko tedaj, ko je $q=2$, in navedemo jih do izomorfizma natančno. Nato dokažemo, da obstajajo regularni uravnoteženi Cayleyevi zemljevidi grupe $M_p(n,m)$ natanko tedaj, ko je $p=2$ in $n=m$ ali $n=m+1$, ter da v obeh izmed teh dveh primerov do izomorfizma natančno obstaja natanko en takšen zemljevid. Nazadnje, kot posledico, dokažemo, da poljubna metaciklična p-grupa za liho praštevilo p nima regularnih uravnoteženih Cayleyevih zemljevidov.

Ključne besede: Regularni uravnoteženi Cayleyev zemljevid, minimalna neabelska grupa, metaciklična grupa.

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