

# $\mathcal{F}$ -WORM colorings of some 2-trees: partition vectors

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## Abstract

Suppose  $\mathcal{F} = \{F_1, \dots, F_t\}$  is a collection of distinct subgraphs of a graph  $G = (V, E)$ . An  $\mathcal{F}$ -WORM coloring of  $G$  is the coloring of its vertices such that no copy of each subgraph  $F_i \in \mathcal{F}$  is monochrome or rainbow. This generalizes the notion of  $F$ -WORM coloring that was introduced recently by W. Goddard, K. Wash, and H. Xu. A (restricted) partition vector  $(\zeta_\alpha, \dots, \zeta_\beta)$  is a sequence whose terms  $\zeta_r$  are the number of  $\mathcal{F}$ -WORM colorings using exactly  $r$  colors, with  $\alpha \leq r \leq \beta$ . The partition vectors of complete graphs and those of some 2-trees are discussed. We show that, although 2-trees admit the same partition vector in classic proper vertex colorings which forbid monochrome  $K_2$ , their partition vectors in  $K_3$ -WORM colorings are different.

*Keywords:* 2-tree, maximal outerplanar, partition, Stirling numbers.

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# $\mathcal{F}$ -WORM barvanja nekaterih 2-dreves: razdelilni vektorji

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## Povzetek

Naj bo  $\mathcal{F} = \{F_1, \dots, F_i\}$  zbirka različnih podgrafov grafa  $G = (V, E)$ .  $\mathcal{F}$ -WORM barvanje grafa  $G$  je takšno barvanje njegovih vozlišč, pri katerem nobena kopija nobenega od podgrafov  $F_i \in \mathcal{F}$  ni ne enobarvna ne mavrična. To je posplošitev pojma  $F$ -WORM barvanja, ki so ga nedavno vpeljali W. Goddard, K. Wash in H. Xu. (Omejen) razdelitveni vektor  $(\zeta_\alpha, \dots, \zeta_\beta)$  je zaporedje, katerega členi  $\zeta_r$  so števila  $\mathcal{F}$ -WORM barvanj z natanko  $r$  barvami, kjer je  $\alpha \leq r \leq \beta$ . Obravnavamo razdelitvene vektorje polnih grafov in nekaterih 2-dreves. Pokažemo, da četudi 2-drevesa premorejo enake razdelitvene vektorje pri klasičnem pravilnem barvanju vozlišč, ki prepoveduje enobarvne  $K_2$ , so njihovi razdelitveni vektorji v  $K_3$ -WORM barvanju lahko različni.

*Ključne besede: 2-drevo, maksimalen zunanjeravninski, razdelitev, Stirlingova števila.*

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