

## Uniformly dissociated graphs\*

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**Abstract:** A set  $D$  of vertices in a graph  $G$  is called a dissociation set if every vertex in  $D$  has at most one neighbor in  $D$ . We call a graph  $G$  uniformly dissociated if all maximal dissociation sets are of the same cardinality. Characterizations of uniformly dissociated graphs with small cardinalities of dissociation sets are proven; in particular, the graphs in which all maximal dissociation sets are of cardinality 2 are the complete graphs on at least two vertices from which possibly a matching is removed, while the graphs in which all maximal dissociation sets are of cardinality 3 are the complements of the  $K_4$ -free geodetic graphs with diameter 2. A general construction by which any graph can be embedded as an induced subgraph of a uniformly dissociated graph is also presented. In the main result we characterize uniformly dissociated graphs with girth at least 7 to be either isomorphic to  $C_7$ , or obtainable from an arbitrary graph  $H$  with girth at least 7 by identifying each vertex of  $H$  with a leaf of a copy of  $P_3$ .

**Keywords:** Dissociation number, well-covered graphs, girth, Moore graph, polarity graph

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## Enakomerno razdruženi grafi\*

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**Povzetek:** Množica  $D$  vozlišč v grafu  $G$  se imenuje razdruževalna množica, če ima vsako vozlišče v  $D$  največ enega soseda v  $D$ . Graf  $G$  imenujemo enakomerno razdružen, če imajo vse maksimalne razdruževalne množice isto moč. Podane so karakterizacije enakomerno razdruženih grafov z majhno močjo razdruževalnih množic; tako so grafi, v katerih imajo vse maksimalne razdruževalne množice moč 2, polni grafi na najmanj dveh vozliščih, iz katerih je izvzeto kvečjemu prirejanje, medtem ko so grafi, v katerih so vse maksimalne razdruževalne množice moči 3, komplementi geodetskih grafov premera 2 brez grafov  $K_4$ . Predstavljena je tudi splošna konstrukcija, s katero poljuben graf lahko vložimo kot induciran podgraf enakomerno razdruženega grafa. V glavnem rezultatu karakteriziramo enakomerno razdružene grafe z ožino najmanj 7 in pokažemo, da so bodisi izomorfnii  $C_7$  bodisi se jih da dobiti iz poljubnega grafa  $H$  ožine najmanj 7 z identifikacijo vsakega vozlišča grafa  $H$  z listom kopije  $P_3$ .

**Ključne besede:** Razdružitevno število, dobro pokriti grafi, ožina, Mooreov graf, polar-nostni graf.

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